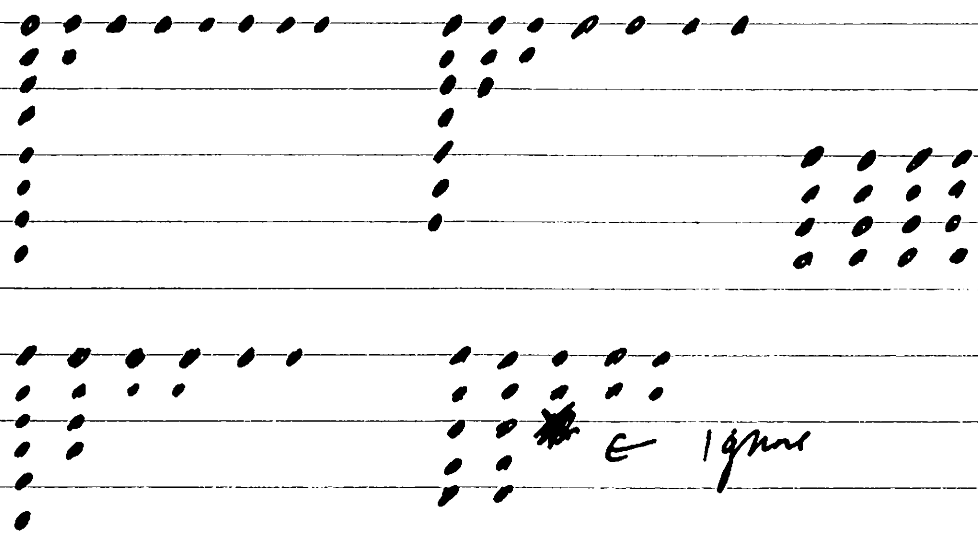
# Problem set 3

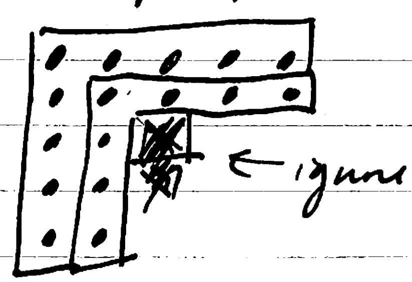
## §2.10

(18) For 16 there are 5; for 19 there are 6. I did this by drawing them. Here is what they look like for 16 (using dots instead of boxes):

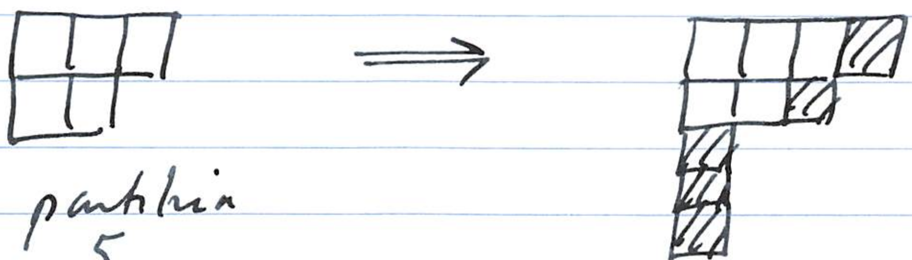



(I started with the upper left and then tried to move ~~boxes~~ dots in from the end.) Incidentally, the number of self-conjugate partitions of $n$ is the same as the number of partitions of $n$ with distinct odd parts. To see it you chop the diagrams into "hooks":

$16 = 9 + 7$



(20) If you try some examples, it looks like the answer is $p(n)$. This is correct. Let $\lambda$ be a partition of $n$. Its diagram has at most $n$ rows. We add $n$ new boxes, one in each row, to make a partition of $2n$ with exactly $n$ parts. (If a row is empty in $\lambda$, we just put a new box there.) We can go backwards from a partition of $2n$ with exactly $n$ parts by deleting a box from each row.



a partition of 5

a partition of 10 with exactly 5 rows.

(22) Given a partition $\lambda$ of $n$, we can make a partition of $n+1$ by adjoining a $\underline{1}$. This means $p(n) \leq p(n+1)$. Any partition of $n+1$ that doesn't have a $\underline{1}$, like $\mu = (n+1)$, can't

be constructed this way. Therefore $p(n) < p(n+1)$.

33 We use I/E. Let $S$ be the set of numbers $\{0, \ldots, 999\}$. Let $A \subset S$ be the multiples of 5 and $B \subset S$ the multiples of 6. Then $A \cap B$ is the multiples of 30. We have

$$|S \setminus A \cup B| = |S| - |A| - |B| + |A \cap B|$$

and

$$|S| = 1000$$
$$|A| = \lfloor 1000/5 \rfloor = 200$$
$$|B| = \lfloor 1000/6 \rfloor = 166$$
$$|A \cap B| = \lfloor 1000/30 \rfloor = 33$$
$$\Rightarrow \quad |S \setminus A \cup B| = \boxed{667}.$$

38 Again we use I/E. Let

$A =$ multiples of 2 that are $\leq 300$ and positive

$B =$ . . . . . . . 3 . . . . . . . . . . . . .

$C =$ . . . . . . . 5 . . . . . . . . . . . . . .

use same method as in 33 to find these orders

We want $|A \cup B \cup C|$. This is

$$|A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C| = 150 + 100 + 60 - 50 - 30 - 20 + 10 = \boxed{220}.$$