DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS

MATH 233H **EXAM 1** **Fall 2014**

YOUR NAME: ____

Section Number: Honors Instructor's Name: Meeks

In problems that require reasoning or algebraic calculation, it is not sufficient just to write the answers. You must explain how you arrived at your answers, and show your algebraic calculations.

You can **leave answers in terms of fractions and square roots**.

$< x, y, z >$, $[x, y, z]$, $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$; are all permissible notations for the vector $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

1.	(10)	______________
2.	(20)	______________
3.	(20)	______________
4.	(15)	______________
5.	(10)	______________
6.	(10)	______________
7.	(15)	______________
Total		______________

Perfect Paper $\longrightarrow$ 100 Points.

There are 9 pages, including this one, in this exam and 8 problems, the last one being extra credit. Make sure you have them all before you begin! There is an additional blank page at the end of the exam if you need more space to write down your solutions.

1. Given $A = (-1, 1, 5)$, $B = (3, 5, 2)$ and $C = (1, 2, 3)$.

a) (5 points) Let L be the line which passes through the points A and B. Find the parametric equations for L.

b) (5 points) A, B and C are three of the four vertices of a parallelogram, while AB and BC are two of its four edges. Find the fourth vertex E of the parallelogram.

2. Consider the points $P(3,1,1)$, $Q(5,1,2)$, $R(5,4,1)$ in $\mathbb{R}^3$.

a1) (10 points) Find an equation for the plane containing P, Q and R.

a2) (5 points) Find the area of the triangle with vertices P, Q, R.

b) (5 points) Find the cosine of the angle between the planes $x + 2y + 2z = 1$ and $-2x + y + 2z = 5$.

3. a) (10 points) Determine parametric equations for the line L tangent to the *space* curve given by $r(t) = \langle 1 + \sin t, \cos t, 2t \rangle$ at $t = \pi/2$.

b) (5 points) Write the parametric equations of the line L containing the point $T(1, 2, 3)$ and perpendicular to the plane $3x + 2y - 2z = 1$.

c) (5 points) Find the point of intersection of the line $r(t) = \langle 1+t, 3t, t \rangle$ with the plane $2x + y - 2z = 8$. (Hint: First find the time t_0 when the line crosses the plane.)

4. a) (10 points) Find the equation of the sphere with center at the point $(1, 2, 4)$ and which contains the point $(0, 4, 6)$.

b) (5 points) Find the center and radius of the sphere $x^2 + 4x + y^2 + 6y + z^2 = 3$. (Hint: Rewrite the surface in standard form by completing the squares.)

5. a) (5 points) Make a sketch of the surface in $\mathbb{R}^3$ described by equation $x = z^2$. In your sketch of this surface, include the labeled coordinate axes and label the trace curves on the surface for the planes $y = 0$ and $y = 4$. (Hint: Notice that there is no restriction on the y-coordinate of this surface.)

b) (5 points) Find the volume **V** of the **parallelepiped** such that the following four points $A = (1, 1, 2)$, $B = (3, 1, -2)$, $C = (1, 3, 2)$, $D = (1, 0, -1)$ are vertices and the vertices B, C, D are all adjacent to the vertex A. (Hint: Use the triple scalar product or determinants.)

6. Let L_1 be the line with parametric equations $x = 1 + 2t$, $y = 2 + t$, $z = 3 - t$, and L_2 be the line with parametric equations $x = 3 + s$, $y = 2 + s$, $z = 1 + s$. There exist unique parallel planes P_1, P_2 with L_1 contained in P_1 and L_2 contained in P_2.

   a) (7 points) Find the equation of the plane P_1.

   b) (3 points) Find the distance between the skew lines L_1, L_2 (which is also equal to the distance between the parallel planes P_1 and P_2, which is also equal to the distance from L_2 to the plane P_1).

7. Suppose $\vec{a} = \langle 2, 2, 1 \rangle$ and $\vec{b} = \langle 3, 6, 0 \rangle$.

a) (10 points) Find the vector projection, call it $\vec{c}$, of $\vec{b}$ in the direction $\vec{a}$.

b) (5 points) Calculate the vector $\vec{b} - \vec{c}$ and then show that it is orthogonal to $\vec{a}$.

8. Suppose $\vec{a} = \langle 2, 2, 1 \rangle$ and $\vec{b} = \langle 8, 2, 0 \rangle$.

a) (10 points) Find the vector projection, call it $\vec{c}$, of $\vec{b}$ in the direction $\vec{a}$.

b) (5 points) Suppose $\vec{a} = \langle 2, 1, 2 \rangle$ and $\vec{b} = \langle 8, 2, 0 \rangle$. Write $\vec{b}$ as a sum of a vector parallel to $\vec{a}$ and a vector orthogonal to $\vec{a}$. (Hint: Use projections.)

.