DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS

MATH 233 **EXAM 1** **Spring 2017**

YOUR NAME: ______________________________________

Check the box next to your section:

- ☐ 1, Andrew Havens, 9:05 - 9:55, MWF
- ☐ 2, Maria Nikolaou, 11:15 - 12:05, MWF
- ☐ 3, Noriyuki Hamada, 12:20 - 1:10, MWF
- ☐ 4, Noriyuki Hamada, 1:25 - 2:15, MWF
- ☐ 5, Dinakar Muthiah, 10:00 - 11:15, TuTh
- ☐ 6, Dinakar Muthiah, 11:30 - 12:45, TuTh
- ☐ 7, Luca Shaffler, 1:00 - 2:15, TuTh
- ☐ 8, Luca Shaffler, 2:30 - 3:45, TuTh
- ☐ 9, William Meeks, 8:30 - 9:45, TuTh
- ☐ 10, Sean Hart 2:30 - 3:45, MW
- ☐ 11, Maria Nikolaou 10:10 - 11:00, MWF
- ☐

In problems that require reasoning or algebraic calculation, it is not sufficient just to write the answers. You must explain how you arrived at your answers, and show your algebraic calculations.

You can **leave answers in terms of fractions and square roots**.

$< x, y, z >$, $[x, y, z]$, $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$; are all permissible notations for the vector $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$.

1.	(15)	________
2.	(15)	________
3.	(15)	________
4.	(15)	________
5.	(15)	________
6.	(15)	________
7.	(10)	________
Total		________

Perfect Paper $\longrightarrow$ 100 Points.

There are 9 pages, including this one and a blank one at the end, in this exam and 7 problems. Make sure you have them all before you begin!

1. In parts a) and b) of this problem consider the points $A = (-1, 1, 3)$, $B = (2, 5, 2)$ and $C = (1, 2, 6)$.

   a) (5 points) Let L_1 be the line which passes through the points A and B. Find the parametric equations for L_1.

   b) (5 points) A, B and C are three of the four vertices of a parallelogram, while AB and BC are two of its four edges. Find the fourth vertex D of the parallelogram.

   c) (5 points) Determine parametric equations for the line L_2 tangent to the *space* curve given by $r(t) = \langle t, 2t^2, 2t \rangle$ at the point $(1, 2, 2)$.

2. Consider the points $P = (3, 1, 1)$, $Q = (4, 1, 2)$, $R = (4, 4, 1)$ in $\mathbb{R}^3$.

a) (10 points) Find an equation for the plane containing P, Q and R.

b) (5 points) Find the area of the triangle with vertices P, Q, R.

3. a) (10 points) Find the parametric equations for the line L_1 of intersection of the planes $x - 2y + z = 6$ and $x + y - z = 0$:

b) (5 points) Find the **cosine** of the angle θ between the planes $x - 2y + z = 6$ and $x + y - z = 0$.

4. a) (10 points) Find the equation of the sphere S with center $(-1, 2, 5)$ and containing the point $(1, 0, 1)$.

b) (5 points) Find the center and radius of the sphere

$$x^2 - 8x + y^2 + 10y + z^2 = 8.$$

5. a) (5 points) Make a sketch of the surface in $\mathbb{R}^3$ described by equation $z = x^2$. In your sketch of this surface, include the labeled coordinate axes and draw and label the trace curves on the surface for the planes $y = 0$ and $y = 3$.

b) (10 points) Find the volume **V** of the **parallelepiped** such that the following four points $A = (2, 1, 2)$, $B = (3, 1, -2)$, $C = (3, 3, 3)$, $D = (2, 0, -1)$ are vertices and the vertices B, C, D are all adjacent to the vertex A. (Hint: Use the scalar triple product or determinants to make this calculation.)

6. a) (10 points) Consider the points $A = (1,1,1)$, $B = (3,3,2)$ and $C = (3,5,16)$. Suppose $\mathbf{a} = \vec{AB}$ and $\mathbf{b} = \vec{AC}$. Find the vector projection, call it $\mathbf{c}$, of $\mathbf{b}$ onto the vector $\mathbf{a}$.

b) (5 points) Calculate the vector $\mathbf{b} - \mathbf{c}$ and then show that this new vector is orthogonal to $\mathbf{a}$.

7. a) (5 points) Suppose that a vector $\vec{v}$ can be written as $\langle 1, 3, k\rangle$, where k is unknown. If $\vec{v}$ is orthogonal to the vector $\langle 15, -12, -7\rangle$, then what is the value of k?

b) (5 points) Write down the parametric equations of the line L containing the point $A = (1, 2, 3)$ and orthogonal (perpendicular) to the plane P defined by

$$x - 2y + z = 6$$