MATH 233 PRACTICE FINAL EXAM, VERSION #2

Important Note: This practice exam is intended to give you an *idea* about what a two-hour final exam is like. It is not possible for any one exam to cover every topic, and the *content, coverage and format of your actual exam could be different from this practice exam.*

PART I: MULTIPLE CHOICE PROBLEMS. You only need to give the answer; no justification is needed.

#I-1. Determine which of the following vector fields is **conservative**.

(a) $\langle -y, x, 0\rangle$ (b) $\langle xy, yz, zx\rangle$ (c) $\langle 2xy, x^2+2yz, y^2\rangle$
(d) $\langle y^2, z^2, x^2\rangle$ (e) $\langle x^2y, y^2z, z^2x\rangle$ (f) $\langle x, x, x\rangle$

#I-2. Let $z = x^2y$, and let x, y be functions of t with $x(1) = 1, y(1) = 2, x(2) = 3, y(2) = 4; x'(1) = A, y'(1) = B, x'(2) = C, y'(2) = D$. Find dz/dt when $t = 1$.

(a) $4A + D$ (b) $4A + B$ (c) $4C + D$ (d) $A + 2D$ (e) $4C + 2D$ (f) $A + 4B$

#I-3. Find the **minimum speed** of the particle whose position function is $\vec{r}(t) = \langle t^2, 1-2t, t\rangle$.

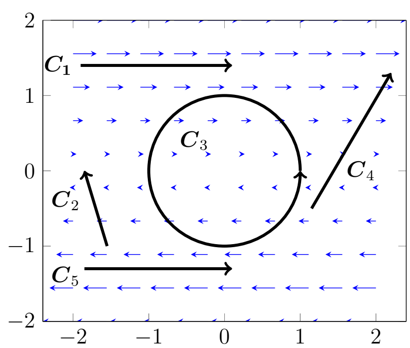
(a) 0 (b) 1 (c) $\sqrt{2}$ (d) $\sqrt{3}$ (e) $\sqrt{5}$ (f) 6

#I-4. Find the value of the **gradient vector field** of the function $z = x^2y^3$ at the point $(1, 1)$.

(a) $\langle 2, -3\rangle$ (b) $\langle 2, 3\rangle$ (c) $\frac{1}{\sqrt{11}}\langle 3, -2\rangle$
(d) $\langle 2, -3, 1\rangle$ (e) $\frac{1}{\sqrt{11}}\langle 2, 3\rangle$ (f) $\frac{1}{\sqrt{14}}\langle 3, -2, 1\rangle$

#I-5. The figure on the right depicts a 2-dimensional vector field $\vec{F}$ as well as five oriented paths. For which of these paths C_i would the value $\int_{C_i} \vec{F}\cdot d\vec{r}$ be **maximum**?

(a) C_1 (b) C_2 (c) C_3 (d) C_4 (e) C_5



#I-6. Find a non-zero vector perpendicular to the plane that passes through the points $(1, 0, 0), (5, 4, 0)$ and $(0, 4, 1)$.

(a) $\langle 1, -4, -5\rangle$ (b) $\langle 1, -4, 5\rangle$ (c) $\langle -5, 1, 4\rangle$
(d) $\langle 4, -4, 20\rangle$ (e) $\langle 4, 1, 5\rangle$ (f) $\langle 4, 4, 20\rangle$

PART II: WRITTEN PROBLEMS. To earn full credit for the following problems **you must show your work.**

You can leave answers in terms of fractions and square roots.

#II-1. Find an equation for the plane consisting of all points that are equally distance from the two points $(1, 1, 0)$ and $(0, 1, 1)$.

#II-2. Compute the line integral $\int_C \vec{F} \bullet d\vec{r}$, where $\vec{F}(x, y) = (y^2 - x^2 y)\vec{i} + xy^2\vec{j}$, and C is made up of three parts: First, the part of the circle of radius 2 going from $(2, 0)$ to $(\sqrt{2}, \sqrt{2})$, followed by the line segment from $(\sqrt{2}, \sqrt{2})$ to the origin, and finally from the origin to $(2, 0)$.

#II-3. Consider the 2-dimensional vector field $\vec{F}(x, y) = e^y \sin x\vec{i} + e^y \cos x\vec{j}$.

(a) Is this vector field **conservative**? If so, find a **potential function** for $\vec{F}$; if not, explain.

(b) Compute the work done by $\vec{F}$ along the line segment from $(0, 0)$ to $(\pi, 1)$.

#II-4. Compute the **mass** of solid bounded by $x = 1 - y^2 - z^2$ and the plane $x = 0$, with density function $\rho(x, y, z) = yz$.

#II-5. Determine the **flux** of the vector field $\boldsymbol{\vec{F} = ye^{z^2}\vec{i} + y^2\vec{j} + e^{xy}\vec{k}}$ across the **cylinder** S defined by $x^2 + y^2 = 9$ and $0 \le z \le 4$, with positive orientation.

#II-6. Compute the surface integral $\iint_S \operatorname{curl}\vec{F} \bullet \vec{n}\, dS$, where $\boldsymbol{\vec{F} = 4z\vec{i} + 3x\vec{j} + 3y\vec{k}}$, and S is the surface given by $z = 10 - x^2 - y^2$ and $z \ge 4$, with positive orientation.

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