MATH 233 PRACTICE FINAL EXAM, VERSION #1

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PART I: MULTIPLE CHOICE PROBLEMS. You only need to give the answer; no justification is needed.

#I-1. Descrie the surface whose equation in **cylindrical coordinates** is $z = 4r$.

(a) **cylinder with vertical axis** (b) **cylinder with horizontal axis** (c) **sphere**
(d) **half cone with vertical axis** (e) **half cone with horizontal axis** (f) **a plane**

#I-2. Which of the following double integrals are such that reversing the order of integration would result in **two double integrals**?

A: $\displaystyle\int_{-1}^{2}\int_{x^2-2}^{x} f(x,y)\,dy\,dx$ B: $\displaystyle\int_{0}^{1}\int_{y^2}^{2-y} g(x,y)\,dx\,dy$ C: $\displaystyle\int_{0}^{1}\int_{\arctan x}^{\pi/4} h(x,y)\,dy\,dx$

(a) **A only** (b) **B only** (c) **C only** (d) **A & B** (e) **A & C** (f) **B & C**

#I-3. Which of the following point is a **local minimum** of the function $xy - x^2y - xy^2$?

(a) $\mathbf{(0,0)}$ (b) $\mathbf{(1,1)}$ (c) $\mathbf{(0,2)}$ (d) $\mathbf{(2,0)}$ (e) $\mathbf{(1/3,1/3)}$ (f) $\mathbf{(1/2,1/2)}$

#I-4. Determine the set on which the function $f(t) = \dfrac{\sqrt{t}-1}{\sqrt{t}+1}$ is **continuous**.

(a) $\boldsymbol{t > 0}$ (b) $\boldsymbol{t \geq 0}$ (c) $\boldsymbol{t > 1}$ (d) $\boldsymbol{t \geq 0}$ (e) $\boldsymbol{t \neq 0}$ (f) **all real numbers**

#I-5. Let $f(x,y,z)$ be a differentiable function, and let $\vec{F}(x,y,z)$ be a differentiable, 3-dimensional vector field. Which of the following formulae is correct?

(a) $\mathrm{div}(\boldsymbol{f\vec{F}}) = \boldsymbol{f}\mathrm{div}\boldsymbol{\vec{F}} + (\mathrm{curl}\boldsymbol{\vec{F}}) \bullet \boldsymbol{\nabla f}$ (b) $\mathrm{div}(\boldsymbol{f\vec{F}}) = \boldsymbol{f}\mathrm{curl}\boldsymbol{\vec{F}} + \boldsymbol{\vec{F}} \times \boldsymbol{\nabla f}$
(c) $\mathrm{curl}(\boldsymbol{f\vec{F}}) = \boldsymbol{f}\mathrm{div}\boldsymbol{\vec{F}} + \boldsymbol{\vec{F}} \bullet \boldsymbol{\nabla f}$ (d) $\mathrm{div}(\boldsymbol{f\vec{F}}) = \boldsymbol{f}\mathrm{div}\boldsymbol{\vec{F}} + \boldsymbol{\vec{F}} \bullet \boldsymbol{\nabla f}$
(e) $\mathrm{curl}(\boldsymbol{f\vec{F}}) = \boldsymbol{f}\mathrm{curl}\boldsymbol{\vec{F}} + (\mathrm{curl}\boldsymbol{\vec{F}}) \times \boldsymbol{\nabla f}$ (f) **none of the above**

#I-6. Let $\vec{v} \in \mathbf{R}^3$ be a **constant, non-zero vector**. Denote by S the surface of the cube with vertices $(\pm1,\pm1,\pm1)$, with **inward-pointing normal vectors**. Compute the surface integral $\displaystyle\iint_S 2\vec{v} \bullet \vec{n} dS$.

(a) $\boldsymbol{-2|\vec{v}|}$ (b) $\boldsymbol{-|\vec{v}|}$ (c) $\mathbf{0}$ (d) $\boldsymbol{|\vec{v}|}$ (e) $\boldsymbol{2|\vec{v}|}$ (f) $\boldsymbol{3|\vec{v}|}$

PART II: WRITTEN PROBLEMS. To earn full credit for the following problems **you must show your work.**

You can leave answers in terms of fractions and square roots.

II-1. Find the point at which the two lines

$$\vec{r_1}(t) = \langle 1,1,0\rangle + t\langle 1,-1,2\rangle, \quad \vec{r_2}(s) = \langle 2,0,2\rangle + s\langle -1,1,0\rangle$$

intersect.

#II-2. Find the work done by the vector field $\vec{F} = x^2y\vec{i} + \frac{1}{3}x^3\vec{j} + xy\vec{k}$ along the curve of intersection C of the paraboloid $z = y^2 - x^2$ and the cylinder $x^2 + y^2 = 1$, oriented counter-clockwise as viewed from the above.

#II-3. Find **every** point on the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$ at which the tangent plane is **parallel** to the plane $3x - y + 3z = 1$.

#II-4. Consider the vector field $\vec{F}(x,y,z) = y\vec{i} + (x+z)\vec{j} + y\vec{k}$.

(a) Is this vector field **conservative**? If so, find a **potential function** for $\vec{F}$; if not, explain.

(b) Compute the line integral of $\vec{F}$ along the line segment from $(2,1,4)$ to $(8,3,-1)$.

#II-5. Find the area of the surface given by the parametric equations

$$\vec{r}(u,v) = \langle uv, u+v, u-v\rangle$$

with $u^2 + v^2 \leq 1$.

#II-6. Compute the **flux** of the fluid field $\vec{F}(x,y,z) = xz\vec{i} + xy\vec{j} + yz\vec{k}$ flowing **into** the surface of the tetrahedron with vertices $(0,0,0), (1,0,0), (0,1,0)$, and $(0,0,1)$,

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