

HOMEWORK 6 (PRACTICE PROBLEMS), HONORS CALCULUS II  
DUE THURSDAY 10/24/2019

Please hand in your home work before class, have it neatly written, organized (the grader will not decipher your notes), stapled, with your name and student ID on top. All integrals have to be computed without using symbolic calculators. You may use a calculator only to verify a result and for numerical calculations which you cannot do on paper or in your head.

**Problem 1.** Calculate the area bounded by the graphs of  $y = f(x)$  and  $y = g(x)$  in the following scenarios (always draw a picture first):

- (i)  $f(x) = \sqrt{x}$  and  $g(x) = x^3$  between their intersections.
- (ii)  $f(x) = e^x$  and  $g(x) = e^{-x}$  over the interval  $[-1, 1]$ .
- (iii)  $f(x) = \ln(2)$  and  $g(x) = \ln(x)$  over the interval  $[\frac{1}{e}, 2]$ .
- (iv)  $f(x) = \frac{\ln(x)}{x}$  and  $g(x) = -x + 1$  from their intersection to  $x = e^2$ .

**Problem 2.** Calculate the following integrals:

- (i)  $\int x^2 e^{-x} dx = ?$
- (ii)  $\int_1^3 x^3 \ln(x) dx = ?$
- (iii)  $\int x \cos(x) dx = ?$
- (iv)  $\int \frac{1}{x^3 - x} dx = ?$
- (v)  $\int_{-1}^1 \frac{x^7 - x^{11}}{\cos x} dx = ?$
- (vi)  $\int_0^{\pi/4} \tan(x) dx = ?$
- (vii)  $\int \frac{(\ln x)^3 + 5}{x} dx = ?$
- (viii)  $\int_0^1 \frac{x^3 + 2x^2 - x + 1}{x + 1} dx = ?$

**Problem 3.** Calculate the lengths of the following curves  $\gamma: [a, b] \rightarrow \mathbb{R}^2$ :

- (i)  $\gamma(t) = (t^2 - 1, t + 1)$  on  $[-1, 2]$ .
- (ii)  $\gamma(t) = (t^2, t^3)$  on  $[0, 1]$ .
- (iii)  $\gamma(t) = t(\cos t, \sin t)$  on  $[0, 2\pi]$ .
- (iv)  $\gamma(t) = (\sin t, (\sin t)^{3/2})$  on  $[0, \pi/2]$ .

**Problem 4.** Determine (and provide a proof) whether the following integrals are finite or not. Explain why the integral is improper: either due to an infinite domain of integration, or due to the integrand having a vertical asymptote at one of the endpoints of the interval; indicate which one it is.

- (i)  $\int_1^\infty (e^{-t^2} + \frac{1}{t^2}) dt$ .
- (ii)  $\int_0^1 \frac{1+x}{\sqrt{x}} dx$ .
- (iii)  $\int_1^\infty \frac{x}{x^3 + 8x^2} dx$ .
- (iv)  $\int_0^\infty \frac{\sqrt{x}}{3x+1} dx$ .

**Problem 5.** Calculate the explicit values of the following integrals. Before you do that, decide whether any of those integrals is an improper integral (either due to an infinite domain of integration, or due to the integrand having a vertical asymptote at one of the endpoints of the interval; indicate which one it is).

- (i)  $\int_0^\infty \frac{dx}{1+e^x}$
- (ii)  $\int_{-5}^0 \frac{x dx}{\sqrt[3]{x+5}}$

$$(iii) \int_{-\pi/2}^{\pi/2} \frac{\sin^3(x)}{1+\cos x} dx.$$

$$(iv) \int_0^1 \frac{dx}{\sqrt{x^2+4x+1}}.$$

$$(v) \int_0^{\ln 3} \frac{e^{2x}}{e^{3x}+e^x} dx.$$