Homework 5, Honors Calculus II
Practice Sheet

Problem 1. Calculate the area bounded by the graphs of $y = f(x)$ and $y = g(x)$ in the following scenarios (always draw a picture first):

(i) $f(x) = \sqrt{x}$ and $g(x) = x^3$ between their intersections.
(ii) $f(x) = e^x$ and $g(x) = e^{-x}$ over the interval $[-1, 1]$.
(iii) $f(x) = \ln(2)$ and $g(x) = \ln(x)$ over the interval $[\frac{1}{e}, 2]$.
(iv) $f(x) = \frac{\ln(x)}{x}$ and $g(x) = -x + 1$ from their intersection to $x = e^2$.

Problem 2. Calculate the following integrals:

(i) $\int x^2 e^{-x}\, dx = ?$
(ii) $\int_1^3 x^3 \ln(x)\, dx = ?$
(iii) $\int x \cos(x)\, dx = ?$
(iv) $\int \frac{1}{x^3 - x}\, dx = ?$
(v) $\int_{-1}^1 \frac{x^7 - x^{11}}{\cos x}\, dx = ?$
(vi) $\int_0^{\pi/4} \tan(x)\, dx = ?$
(vii) $\int \frac{(\ln x)^3 + 5}{x}\, dx = ?$
(viii) $\int_0^1 \frac{x^3 + 2x^2 - x + 1}{x + 1}\, dx = ?$

Problem 3. Calculate the lengths of the following curves $\gamma\colon [a, b] \to \mathbb{R}^2$:

(i) $\gamma(t) = (t^2 - 1, t + 1)$ on $[-1, 2]$.
(ii) $\gamma(t) = (t^2, t^3)$ on $[0, 1]$.
(iii) $\gamma(t) = t(\cos t, \sin t)$ on $[0, 2\pi]$.
(iv) $\gamma(t) = (\sin t, (\sin t)^{3/2})$ on $[0, \pi/2]$.

Problem 4. Calculate the surface area and volume of the funnel formed by revolving $y = e^{-x}$, $x \in [0, b]$, around the x-axis (draw a picture). What happens to the area and volume if $b \to \infty$?

Problem 5. Calculate the volume of the solid obtained by revolving the region between the graphs of $y = e^x$ and $y = e^{-x}$ around the x-axis for $0 \leq x \leq \ln(4)$ (draw a picture).

Problem 6. Calculate the surface area and volume of the solid generated by revolving the region between the graphs $y = \sqrt{x}$ and $y = x$ around the y-axis (draw a picture).

Problem 7. Determine (and provide a proof) whether the following integrals are finite or not:

(i) $\int_0^\infty e^{-t^2}\, dt$. Think of area comparison to some area you know something about...
(ii) $\int_0^1 \frac{1}{x}\, dx$
(iii) $\int_1^\infty \frac{2x}{x^3 + 8}\, dx$. Look at HW 5.
(iv) $\int_0^\infty \frac{\sqrt{x}}{x + 9}\, dx$. It is true that eventually $x + 9 \leq 9x$ (why? Draw the two graphs to understand this statement), which should somehow help...think of area comparison...