Assessment sheet, Honors Calculus II
Fall 2019

The following problems are meant to (self)assess your abilities of basic high school mathematics and college level Calculus I. Nevertheless, you should have not too many difficulties doing them. If you encounter difficulties discuss them with your peers first. Even if initially you cannot solve all the problems to your satisfaction, as long as you have the passion to understand mathematics at a deeper level—and not just to earn a passing grade—you will be fine, I think.

Problem 1. Consider a regular n-gon in the plane. Choose one vertex of the n-gon and draw lines from this vertex to all of the other vertices. Is it true that the angles between those lines passing through the chosen vertex are all equal? You will need to provide a proof/demonstration of your answer, a guess is not considered an answer. To get an idea about the situation, you may want to consider first what happens for a 3-gon and a 4-gon. Sometimes special cases provide ideas how to treat the general case. Play with it.

Problem 2. Consider a regular n-gon whose vertices lie on a circle of radius one. Try to find a formula for the circumference of such an n-gon. If you do not succeed to do this for a general integer n, do the exercise first for $n = 3, 4, 5, 6, 7$. You notice that this is rather tedious, since you have to calculate each case separately—thus the need for a general formula (mathematicians do not like to endlessly do special cases once they see a pattern for the general case—life is too short).

Can you explain what this has to do with finding a value for the circumference of a circle of radius one?

Problem 3. Consider the equation

$$y^2 = x(x+1)(x-1)$$

whose solutions carve out a curve in the plane. Try to draw a fairly accurate picture (without using a calculator!) of this curve by calculating

- (i) The points on the curve where the tangent lines are horizontal (how many such points are there?)
- (ii) The points on the curve where the tangent lines are vertical (how many such points are there?)
- (iii) The inflection points on this curve; regions where the curve is concave up/down; the behavior of the curve when x becomes very large in the positive and negative direction.

Problem 4. Imagine you are in an airport hallway which has a conveyor belt for pedestrians of length L meters and some moving speed v_c meters per second. Now consider the following two scenarios:

- (i) A person walks next to the conveyor belt (so the person is NOT on the conveyor belt!) down the whole length of the conveyor belt and back to its beginning with a constant speed v_p.
- (ii) The same person walks ON the conveyor belt (in the direction of its movement) to its end and walks back ON the conveyor belt to its beginning with the same speed v_p.

Here the question: does it take the person the same amount of time to walk in either scenario, or is there a difference in the time it takes between the two scenarios? If the latter is the case, how much is the time difference? There is a thought experiment to decide the first part of the question without any calculations....