DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS
MATH 131, Fall 2009
Final Exam (Jan 10 makeup)

Your Name: ______________________________

Circle Your Instructor/Section:

Lian (A) MWF 9:05 - 9:55 am	Lian (B) MWF 10:10 - 11:00 am
Herr (C) MWF 11:15 - 12:05 pm	Friedlander (D) MWF 12:20 - 1:10 pm
Koonz (E) MWF 1:25 - 2:15	Del Campo (F) MWF 2:30 - 3:20 pm
Benincasa (G) MW 3:35 - 4:50 pm	Chen (I) TTh 9:30 - 10:45 am
List (J) TTh 11:15 - 12:30 pm	Sommers (K) TTh 1-2:15 pm
Mirkovic (L) TTh 2:30 - 3:45	Gagnon (P) TTh 1- 2:15 pm
McGibbon (R) MWF 11:15 - 12:05	Shumway (S) MWF 12:20 - 1:10 pm
Worth (T) TTh 11:15 - 12:30 pm	

The exam consists of 5 multiple choice questions and 6 long answers. Each problem is worth the indicated number of points. On this exam you may use a page of your own **handwritten** notes, but no books and **no** calculators. The last page is left blank for you to use as scrap paper.

There are multiple versions of the exam, so please do not try to look on a neighbor's paper. Turn off and do not attempt to use your cellphone, iPod, etc at anytime during the exam.

You do not need to justify your answers to the multiple choice questions, but you must *show your work* in the exam booklet to receive credit for the long answer problems. Partial credit will be awarded based on your work. Be prepared to show your UMass ID when you turn in the exam.

1-5: (25 points) ____________
6: (20 points) ____________
7: (20 points) ____________
8: (25 points) ____________
9: (20 points) ____________
10: (20 points) ____________
11: (20 points) ____________
TOTAL: (150) ____________

Multiple choice. Circle the correct answer. (5 points each).

1. The radius of a circle is increasing at a rate of 3 centimeters per second. How fast is its area increasing when the radius is 2 cm?

   (a) $36\pi \ \frac{\text{cm}^2}{\text{s}}$

   (b) $18\pi \ \frac{\text{cm}^2}{\text{s}}$

   (c) $12\pi \ \frac{\text{cm}^2}{\text{s}}$

   (d) $6\pi \ \frac{\text{cm}^2}{\text{s}}$

   (e) $4\pi \ \frac{\text{cm}^2}{\text{s}}$

2. A function $f(x)$ whose second derivative exists everywhere satisfies $f'(5) = 0$ and $f''(5) = 1$. Also $f(x)$ has only **one** critical point. Then

   (a) $f(x)$ has an inflection point at $x = 5$.

   (b) $f(x)$ is increasing for $x < 5$.

   (c) $f(x)$ is concave down at $x = 5$.

   (d) $f(x)$ has a local maximum at $x = 5$.

   (e) $f(x)$ has an absolute minimum at $x = 5$.

3. For which value of a does the tangent line to the curve $y = f(x) = xe^x$ at the point $(a, f(a))$ have a slope that is minimum among all possible choices for a?

   (a) $x = 0$

   (b) $x = -2$

   (c) $x = -1$

   (d) $x = 1$

   (e) $x = e^{-1}$

4. A function $f(x)$ has a **derivative** $f'(x) = (x-1)^c$, where c is a positive integer. Which of the following is **NOT** true?

   (a) $f(x)$ has an inflection point when $x = 1$ if c is 2009.
   (b) $f(x)$ has an inflection point when $x = 1$ if c is 2010.
   (c) $f(x)$ has a local minimum when $x = 1$ if c is 1961 (Obama born)
   (d) $f(x)$ has an absolute minimum when $x = 1$ if c is 1863 (UMass founded).
   (e) $f(x)$ has only one critical number if c is the year you were born.

5. The everywhere differentiable function $f(x)$ satisfies $f(0) = 2$ and $-1 \leq f'(x) \leq 2$. Then using the Mean-Value Theorem, we know that

   (a) $-1 \leq f(5) \leq 2$
   (b) $-2 \leq f(5) \leq 1$
   (c) $-3 \leq f(5) \leq 12$
   (d) $-4 \leq f(5) \leq 11$
   (e) $-5 \leq f(5) \leq 10$

Long answer. Show all work necessary to justify your answers.

6. (20 points) Let $f(x) = 2x^3 - 9x^2 - 60x + 4$. Using methods of calculus,

   (a) (5 points) Find the critical numbers of f.

   (b) (10 points) Classify the critical numbers as local maximum or minimum or neither. Please indicate whether you are using the **first derivative test**, the **second derivative test** or some other method.

   (c) (5 points) Find the interval(s) where f is increasing and decreasing.

7. (20 points) The **velocity** of a particle moving along the x-axis obeys the equation:

$$v(t) = 6\cos(\pi t)$$

where the units of v are meters per second. Consider the motion of the particle for $0 \le t \le 2$.

(a) When is the particle at rest during these 2 seconds? (5 points)

(b) If the particle is located at $x = 0$ at $t = \frac{5}{6}$, find a formula for the position $x = s(t)$ of the particle on the x-axis as a function of time. (7 points)

(c) How much distance does the particle traverse over these 2 seconds? (8 points)

8. Norm Abrams is building a box whose base length is 2 times the base width. The material used to build the top and bottom costs \$10 per square foot and the material used to build the four sides costs \$6 per square foot. If the box must have a volume of 50 cubic feet, determine the dimensions that will minimize the cost to build the box. (25 points)

9. Evaluate the limits.

(a) $\lim_{x \to 0} \frac{\tan x}{4x}$ (5 points)

(b) $\lim_{x \to \pi/4} \frac{\tan x}{4x}$ (5 points)

(c) $\lim_{x \to \infty} (1 + \frac{5}{x})^x$ (10 points)

10. (20 points)

(a) Approximate the definite integral

$$\int_1^5 \frac{1}{2+x}\,dx$$

by using $n = 8$ subdivisions of the interval and **left** endpoints. Set up the sum, but do not evaluate it.

(b) Compute the exact value of the definite integral

$$\int_1^5 (6-3x)\,dx$$

by graphing the function and then thinking about the various areas involved.

11. (20 points) Let

$$f(x) = \frac{x^2}{x^2+1}.$$

(a) On which intervals is $f(x)$ concave up?

(b) What are the inflection points of $f(x)$? (be sure to explain why these are actual inflection points).

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