REU Conference Abstracts

July 23, 2019

Quantum Graph Entropy and the Phase Space visualization of Schrodinger's equation

Speakers: Kitty Girjau, Andrew Moore and Dawit Wachelo
Supervisor: Ivan Contreras (Amherst College)

Abstract: Von Neumann Graph Entropy (VNGE) is a measure of the complexity of a graph calculated using the spectrum of the graph Laplacian. Here, we shall present a brief overview of the underlying theory, some proven results and various approximation methods for VNGE. We then discuss the phase space representation of the Schrodinger equation for graphs and its visualization, as well as ongoing work on the periodicity of the solutions.

Graph Topology and Discrete Morse Theory

Speakers: Andrew Rosevear and Andrew Tawfeek
Supervisor: Ivan Contreras (Amherst College)

Abstract: One can adapt the tools of algebraic and differential topology to study topological properties of the graph, such as number of cycles and connected components. We use the graph chain complex and homology to motivate and define the even and odd graph Laplacians and graph de Rham calculus. We then present discrete Morse theory on a graph, including an equivalence class of Morse functions that gives rise to a simple proof of the Morse inequalities. Further directions include generalization to CW complexes, application of discrete differential forms to problems in graph quantum mechanics, and finding a closed form expression for the number of Morse equivalence classes on a given graph.

An Invariant of 3-colored Knots

Speakers: Olivia Del Guercio, Jack Kendrick, and Hana Sambora
Supervisor: Patricia Cahn (Smith)

Abstract: In this work we compute a knot invariant known as the dihedral linking number for all 3-colorable knots up to 12 crossings. Generally, a linking number is used to describe how many times two distinct knots wind around each other. When a knot has a valid 3-coloring, it lifts to two knots in a separate three-dimensional space known as a branched cover. The dihedral linking number is the linking number of these two knots. This number could potentially allow for the differentiation of mutant knots. We are also looking into the dihedral linking numbers produced by coloring knots with any prime number of colors, which will allow for analysis of a greater number of knots.

Modeling Population Distributions and Spatial-Temporal Patterns of Animal Groups with Producer-Scrounger Behavior

Speakers: Caira Anderson and Issa Susa
Supervisor: Nessy Tania (Smith)

Abstract: Among many animal species, groups form for protection, hunting, and foraging. Patterns emerge from social interactions between the different animals within these groups. In animal groups, some individuals, known as "producers", search for food (prey) on their own, while others, known as "scroungers", exploit the producers. This "producer-scrounger" behavior is common in patchy environments where resources are limited. This interaction results in scroungers not receiving as much food as the producers. We will build a mathematical model consisting of partial differential equations that tracks the population sizes of the producers, scroungers, and their prey. We will study the long-term population distributions due to birth, death, and competition, as well as spatial-temporal patterns that emerge from the model.

Stability of Strip for 2D Incompressible Fluid

Speakers: Victoria Camarena, Christian Madrigal, and Sasha Shrouder
Supervisor: Jennifer Beichman (Smith)

Abstract: Vortices in 2-Dimensional incompressible fluids modeled by Euler equations with simple shapes such as circles and ellipses are typically stable. Similarly, Beichman and Denisov (2017) showed that a rectangular strip is stable in the periodic domain 0 to 2π. We will alter the boundary of the strip in the periodic domain, and we will share observations as to how the stability of the vortex changes. We will also observe whether alterations to the strip return to the steady-state strip.

Oblivious Points on Translation Surfaces

Speakers: Krish Desai, Anthony Ji, Grace Zdeblick
Supervisor: Ian Adelstein and Aaron Calderon (Yale)

Abstract: The study of translation surfaces, collections of polygons in the plane with edges identified by translation, is closely related to billiard trajectories and illumination problems. An oblivious point is a point on a translation surface with no closed geodesics through it. Nguyen, Pan, and Su (2018) showed that there are only finitely many oblivious points on any given translation surface and constructed a surface with exactly one oblivious point. We extend this result to restrict explicitly which points can be oblivious on surfaces tiled by a regular polygon. We also examine the relationship between covering maps, closed geodesics, and illumination to construct a new family of translation surfaces with arbitrarily many oblivious points. Additionally, we devise a new effective algorithm to determine if a point is oblivious using dynamics and combinatorial techniques

Multigraph Chord Diagrams

Speakers: Mrinal Dursun, Joshua Hinman, Logan Rumbaugh
Supervisor: David Freund (Yale)

Abstract: A multistring is a set of closed curves on an oriented 2-manifold, considered up to orientation-preserving homeomorphism. A virtual multistring is an equivalence class of multistrings wherein we can add and remove handles from the surface without disturbing the curves. There is an established combinatorial model for virtual multistrings called a chord diagram. In this talk, we introduce multigraph chord diagrams, combinatorial models for multistrings that capture information about the intersections of a multistring and the surface on which it lives. We will discuss the potential strengths and weaknesses of using multigraph chord diagrams to address problems such as distinguishing between collections of closed curves on surfaces.

On the Erdős Conjecture for Primitive Sets in Function Fields

Speakers: Andres Gomez-Colunga, Charlotte Kavaler, Mirilla Zhu
Supervisor: Nathan McNew (Yale)

Abstract: A primitive set is one in which no element divides another. In 1935, Erdős proved that $f(A) = \sum_{a\in A} \frac{1}{a\log a}$ converges for any primitive set A and later conjectured that this sum is maximized when A is the set of primes. Banks and Martin further conjectured that $f(\mathcal{P}_1) > \ldots > f(\mathcal{P}_k) > f(\mathcal{P}_{k+1}) > \ldots$, where $\mathcal{P}_j$ denotes the set of naturals with j prime factors counting multiplicity. Earlier this year, Bayless et al. showed that $f(\mathcal{P}_1) > f(\mathcal{P}_2) > f(\mathcal{P}_3)$, but the rest of the conjecture remains open.

We consider the corresponding problem over the function field $\mathbb{F}_q[x]$, investigating the sum $f_q(A) = \sum_{f(x)\in A} \frac{1}{\deg f \cdot q^{\deg f}}$. We show that this sum converges when A is a primitive set of polynomials and conjecture that it is maximized by the set of monic irreducibles. Letting $\mathcal{I}_j$ denote the set of monic polynomials with j irreducible factors, we demonstrate that each $f_q(\mathcal{I}_j)$ approaches $\zeta(j+1)$ as q increases and use this to establish the chain of inequalities

$$f_q(\mathcal{I}_1) > \ldots > f_q(\mathcal{I}_k) > f_q(\mathcal{I}_{k+1}) > \ldots$$

for all sufficiently large q. We anticipate that this result will be a useful step towards proving the analogue of Erdős' conjecture in the function field case.

Graph Spectral Denoising for Placenta Images

Speakers: Andrew Benz, Lilly Gold, Rodrigo Ferreira da Rosa, Neha Verma
Supervisor: Karamatou Yacoubou Djima (Yale)

Abstract: In this work, we develop a new graph spectral denoising technique with the goal of analyzing images depicting the placental chorionic surface vascular network (PCSVN). In graph image denoising, images are often represented by an expansion in terms of the eigenvectors of a weighted graph Laplacian. The edge weights are defined via a kernel designed to capture intrinsic structure in an image. We propose a new kernel that emphasizes the similarity between two pixels based on their response to directional wavelets at different scales and shear orientations. This choice is based on the presence of strong directional and self-similar behavior of the vessels in our image. We expect that since this graph construction encapsulates more information about the structure of the image, it results in better spectral denoising of our images.

Investigating skin pattern formation in zebrafish

Speakers: Addie Harrison, Gisela Hoxha, Gil Parnon, Madison Russell, Berke Turkay
Supervisor: Bjorn Standstede (Brown)

Abstract: Zebrafish (Danio rerio) are fish that live in freshwater and have black stripes and yellow interstripes. Zebrafish are an interesting species to study because their skin patterns are highly complex and involve many different variables can be modeled from a mathematical perspective. Many of the mechanisms involved in determining skin patterns in zebrafish are still not well understood, but recent empirical research on zebrafish patterns provides us with new ways to model skin patterns in zebrafish more accurately. Studying the development of skin patters in zebrafish has significant implications in developmental biology, cancer and genetic diseases.

Resistance scaling on the Octacarpet

Speakers: Claire Canner, William Huang, Michael Orwin
Supervisor: Luke Rogers and Chris Hayes (UConn)

Abstract: The study of analytic structures on self-similar fractal sets was initiated by physicists who discovered that heat flow on such sets had sub-Gaussian rather than Gaussian scaling, indicating that the fundamental physics of these sets was very different than on manifolds. These results were first made rigorous for sets with a finite ramification property, but in the late 1980s Barlow and Bass developed a corresponding theory on a class of generalized Sierpinski carpets. Their approach depends on taking a (weak) limit of Brownian motions on a suitable sequence of closed sets that intersect to the carpet. A key step in proving that the limiting object has sub-Gaussian scaling is showing that the resistance of the approximating domain of scale n is bounded above and below by ρ^n for a factor ρ that depends on the carpet. Computing the exact value of ρ remains an open problem.

We consider the resistance scaling problem for the octacarpet, and more generally for $4N$-carpets, with the goal of showing analogous bounds for the resistance and obtaining numerical estimates for the resistance scaling factors.

Decimation structure of the spectra of self-similar groups

Speakers: Brett Hungar, Madison Phelps, Johnathan Wheeler
Supervisor: Luke Rogers and Gamal Mograby (UConn)

Abstract: Physicists and mathematicians have used the self-similar nature of certain fractals to develop and study analytical structures on fractal spaces. We examine the analytical structure of a class of fractals that arise as limit sets of the Schreier graphs of the action of self-similar groups on infinite n-ary trees. In particular, we consider how the spectrum of a Laplacian operator on one level of a Schreier graph relates to the spectrum on the next level, a technique known as spectral decimation.

Grigorchuk and collaborators have developed a method to spectrally decimate Schreier graphs of several important self-similar groups, and have derived significant consequences about the structure of amenable groups. Their method is related to a notion of spectral similarity arising from the work of Fukushima-Shima and Malozemov-Teplyaev. In the latter a sufficient condition for spectral decimation for fractal graphs is obtained. We consider the analogous question for Schreier graphs of self-similar groups with the goal of understanding the class to which Grigorchuk's approach is applicable.

Can we hear the shape of a fractal? Spectral analysis of self-similar sets

Speakers: Elizabeth Melville, Nikhil Nagabandi
Supervisor: Luke Rogers and Gamal Mograby (UConn)

Abstract: Analytic structures on fractals have been analyzed extensively in the past 50 years both because of their interesting mathematical properties and their potential applications in physics. One important question in this area is how the spectrum of a Laplacian on a fractal reflects its geometry; one version of the corresponding problem for domains in Euclidean space was famously described in Kac's question "Can you hear the shape of a drum?".

Some features of the spectra of self-similar sets, such as the asymptotic behavior of the eigenvalue counting function, can be obtained using renewal theory (as in the work of Kigami-Lapidus), but our interest is in more precise results that give the locations and multiplicities of eigenvalues explicitly. These are connected to a long strand of research in mathematical physics about the structure of spectra of Schrödinger operators and their relation to toplogical invariants of the underlying space (prominent results in this area are due to Landau, Peierls, Harper, Moser, Bellissard, and, recently, Avila and Jitomirskaya). One name for these results is gap-labeling theorems. For certain highly-symmetric self-similar sets the computation of the gap structure of the Laplacian spectrum is possible using spectral decimation. We use this method to explicitly compute the gap structure for the Laplacian on a particular two-point self-similar graph and its fractal limit, and for Sierpinski graphs and the Sierpinski gasket.

Hedging by Sequential Regression in Generalized Discrete Models and the Follmer-Schweizer decomposition

Speakers: Sarah Boese, Tracy Cui, Sam Johnston
Supervisor: Oleksii Mostovyi and Gianmarco Molino (UConn)

Abstract: We consider the problem of optimal hedging in the binomial and trinomial pricing models. Using the discrete-time Follmer-Schweizer decomposition, we demonstrate the equivalence of the backward recursion and sequential regression approaches in the binomial case. We conclude with examples of numerical hedging trials in the binomial model. Our continuing work is focused on extending the sequential regression to the incomplete trinomial model.

Distribution of Missing Sums in Correlated Sumsets

Speakers: Thomas Martinez, Dylan King and Chenyang Sun
Supervisor: Steven Miller (Williams)

Abstract: For any finite sets of integers A,B, define the sumset A+B to be $\{a+b \mid a, \in A,\ b \in B\}$. Given a 3-tuple of probabilities (p, p_1, p_2) where $p, p_1, p_2 \in [0,1]$, we examine the random variable $|A+B|$, where for each $i, \mathbf{P}(i \in A) = p, \mathbf{P}(i \in B \mid i \in A) = p_1$, and $\mathbf{P}(i \in B \mid i \notin A) = p_2$. The case where $p = 1/2, p_1 = 1$, and $p_2 = 0$ forces B to equal A, and is well studied. In a recent paper, Lazarev, Miller, and O'Bryant computed the expected value and variance of $|A+A|$ and also $m(k)$, the probability that k sums are missing from $|A+A|$. They provide exponential upper and lower bounds on $m(k)$; their major ingredient in doing so is a powerful graph-theoretic framework expressing $\mathbf{P}(i, j \notin A+A)$ using the Fibonacci numbers. Combining their bounds on m(k) with large-scale computation, they were able to prove the existence of a "divot" in the probability of missing exactly 7 sums; that $m(7) < m(6) < m(8)$.

We extend the graph-theoretic framework to accommodate any (p, p_1, p_2). The primary difficulty is that we replace the second set A with correlated B, dependent upon the structure of A. We overcome this difficulty by first classifying dependent components and then solving a vertex covering problem on these

components using a system of recurrence relations. Using this, we generalize many results from Lazarev, Miller, and O'Bryant, such as bounds on $m(k)$, $\mathbf{E}[|A+A|]$, Var$[|A+A|]$, and we then demonstrate how our framework copes with correlated sets B with given p_1, p_2.

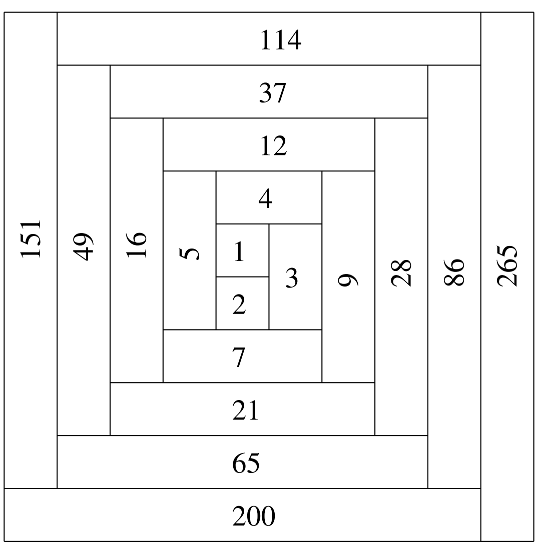
Fibonacci Quilt Game

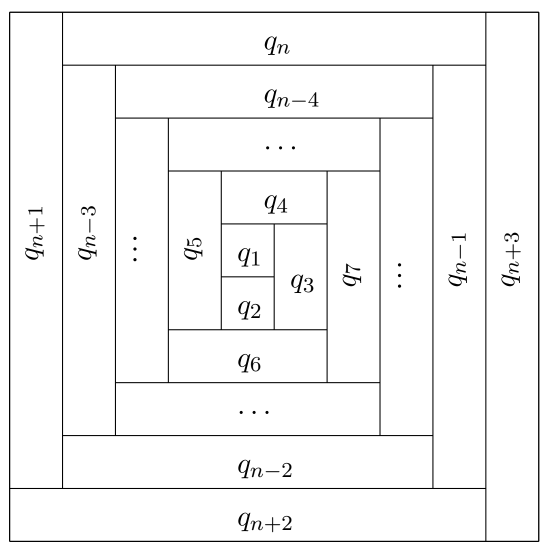
Speakers: Alexandra Newlon, Neelima Borade, Annie Xu, Catherine Wahlenmayer
Supervisor: Steven Miller (Williams)

Abstract: Zeckendorf proved that every positive integer has a unique representation as a sum of non-consecutive Fibonacci numbers; conversely, an equivalent definition of the Fibonacci numbers is that they are the unique sequence such that every positive integer can be uniquely written as a sum of non-adjacent terms (Here, we begin the Fibonacci sequence with 1, 2, 3, 5, ...). This is the first of many interplays between notions of legal decompositions and definitions of a sequence, expanded to a large class of linear recurrences. We can use these notions to create interesting games. The first, the Zeckendorf game, was defined by Baird-Smith, Epstein, Flint, and Miller based on the recurrence relation of the Fibonacci sequence $\{F_n\}$. The Zeckendorf game on an integer n begins with n 1's in bin F_1 and bins $F_2, F_3, \ldots$ empty. Two players then take turns applying the following rules:

- $F_i + F_{i+1} \longrightarrow F_{i+2}$
- $2F_i \longrightarrow F_{i-2} + F_{i+1}$

where F_i is the i^{th} Fibonacci number. It was proved that a Zeckendorf game always terminates at a legal decomposition. If $n \neq 2$ then Player Two has a winning strategy, though the strategy is unknown as the proof is non-constructive. It is also conjectured that game length tends to a Gaussian distribution. We adapt this game to a different sequence, the Fibonacci Quilt sequence, as defined by Catral, Ford, Harris, Miller, and Nelson. The sequence is constructed by adding the next integer which cannot be expressed as the sum of non-adjacent tiles of a log cabin quilt, starting with 1 at the center, shown below:





Legal decomposition for this sequence, referred to as FQ-legal, requires the same non-adjacent quilt behavior. By construction, each integer has an FQ-legal decomposition, but it is not unique. The sequence is eventually dictated by two recurrence relations

$$q_{n+1} = q_n + q_{n-4}, n \geq 6$$

$$q_{n+1} = q_{n-1} + q_{n-2}, n \geq 5,$$

from which we define four rules with additional initial moves. Using a mono-variant, the sum of the square roots of the indices on each move, we prove that under our construction the game always terminates in an FQ-legal decomposition. We investigate the distribution of the length of a random game, as well as bounds on the game length. We also examine deterministic games and the existence of a potential winning strategy.

Generalizing Zeckendorf's Theorem to Homogeneous Linear Recurrences

Speakers: Clayton Mizgerd, Thomas Martinez and Chenyang Sun
Supervisor: Steven Miller (Williams)

Abstract: Zeckendorf's theorem states that every positive integer can be written uniquely as the sum of non-consecutive Fibonacci numbers, where we take $F_1 = 1$ and $F_2 = 2$; in fact, it provides an alternative definition of the Fibonacci numbers. This has been generalized for any Positive Linear Recurrence Sequence (PLRS), which is, informally, a homogeneous linear recurrence with a positive leading coefficient and non-negative integer coefficients; the previous results crucially depend on these conditions to avoid technical issues with their definition on what decompositions are legal, as otherwise we can lose uniqueness of decompositions. We search for these legal decompositions as they are generalizations of base B decompositions. We further this investigation to linear recurrences with leading coefficient zero, followed by non-negative integer coefficients, with differences between indices relatively prime (abbreviated ZLRS), via two different approaches.

The first approach involves generalizing the definition of a legal decomposition for a PLRS found in Koloğlu, Kopp, Miller and Wang. Focusing on ZLRS's of the form $a_n = \alpha a_{n-2} + \beta a_{n-3}$, where $\alpha, \beta \in \mathbb{Z}^+$, we prove that every positive integer N has a legal decomposition using the greedy algorithm. We also show that d_n, the number of decompositions, grows faster than the terms of a_n, implying that existence of decompositions for every positive integer N, but uniqueness is lost.

The second approach instead converts a ZLRS to a PLRS that has the same growth rate and yields the same sequence, given suitable initial values. We have constructed an algorithm for the conversion in constant space and linear run-time in the order of the characteristic polynomial of the PLRS produced, and proved that in many cases it terminates and converts a ZLRS to a PLRS; we are currently working to prove that this algorithm terminates for any ZLRS. We demonstrate how this algorithm works, and illustrate with the examples $a_n = a_{n-2} + a_{n-3}$ and $a_n = a_{n-2} + 2a_{n-3} + a_{n-5}$.

Crescent Configurations In Non-Euclidean Norms

Speakers: Sara Fish, Dylan King, Catherine Wahlenmayer
Supervisor: Steven Miller (Williams College), Evindur Palsson (Virginia Tech)

Abstract: We investigate a generalization of a distinct distances problem first posed by Erdős in 1989. Consider the set of distinct distances determined by n points in the normed metric space $(\mathbb{R}^2, d)$. We say that n points in the plane in general position form a $(\mathbb{R}^2, d)$ *crescent configuration* if for all $1 \leq i \leq n-1$, there is a distance which occurs exactly i times. Erdős asked for which $n \in \mathbf{N}$ there exist $(\mathbb{R}^2, L^2)$ crescent configurations on n points. Constructions for $n \leq 8$ have been found, but this question remains open for $n \geq 9$.

Since the geometry of a space is heavily dependent upon the norm with which it is equipped, it is natural to ask the same question for various concepts of distance. We explore the properties of $(\mathbb{R}^2, L^p)$ crescent configurations for $1 \leq p \leq \infty$. In this non-Euclidean setting, we provide explicit constructions of crescent configurations and adjust the notion of general position so that the existence of such crescent configurations is non-trivial.

ABBA and the Random Matrix Discotheque

Speakers: Keller Blackwell and Wanqiao Xu (Joint work with Neelima Borade, Charles Devlin VI, Renyuan Ma, and Leticia Mattos da Silva)
Supervisor: Steven Miller (Williams)

Abstract: Random Matrix Theory (RMT) is uniquely suited to the fundamental problem of studying the spacing of observed values from complex systems. Montgomerys 1973 pair correlation conjecture hinted at the limiting behavior of spacings between zeros of the Riemann zeta function. When shown to Dyson, he observed that the conjecture matched the distribution of eigenvalues from certain random matrix ensembles, framing the entrance of RMT into number theory as a fresh approach to the zero behavior of the Riemann zeta function and, more generally, L-functions.

RMT successfully models many properties of L-functions; however, there are situations where it is silent. One instance is the number theory process of Rankin-Selberg convolution, which creates a new L-function from an input pair. Our work investigates a possible RMT analogue of this process through the parallel study of random matrix ensembles constructed from existing families.

Let A be a random symmetric Toeplitz matrix with a palindromic first row and B denote a random real symmetric matrix; it is well-known these ensembles converge to the Gaussian and semi-circle distribution, respectively. We consider the "disco" $\mathcal{D}(A,B) = \begin{bmatrix} A & B \\ B & A \end{bmatrix}$. While any distributions may be input, our choices have eigenvalue densities at polar extremes. Computing the $2k^{\text{th}}$ moment of these distributions involves the combinatorial problem of pairing $2k$ items on a circle. If every pairing contributes fully one gets a Gaussian, while if only non-crossing pairs contribute one gets the semi-circle. The hybrid family of random matrices converges to a new, universal distribution of unbounded support.

We use the Eigenvalue Trace Lemma to derive the limiting spectral measure of the eigenvalues of $\mathcal{D}(A,B)$, as we randomly choose A and B, through the formula $\mathbb{E}\left[\text{tr}(\mathcal{D}(A,B)^k)\right] = \mathbb{E}\left[\text{tr}((A+B)^k + (A-B)^k)\right]$. The primary obstacle, which is not present in other random matrix investigations, is analysis of the non-commutative matrix polynomial from the trace expansion

$$\mathbb{E}\left[\text{tr}\left(A^{h_1}B^{j_1}\cdots A^{h_p}B^{j_p}\right)\right] = \sum_{1\le i_1,\cdots,i_k\le N} \mathbb{E}\left[a_{i_1,i_2}\cdots b_{i_k,i_1}\right]$$

for which summands contribute in the limit $N \to \infty$ only if the a's and b's are appropriately paired. An adaptation of the method of moments reformulates such analysis as a combinatorial problem of independent interest. We derive an explicit formula for this combinatorial sum, prove convergence of the hybrid distribution, show it is sharply bounded away from the constituent distributions, and discuss potential applications to open inquiries in number theory.

Spies and Traitors: Random Matrix Kaleidoscopes and their Turncoat Eigenvalues

Speakers: Neelima Borade and Renyuan Ma
Supervisor:

Abstract: Random Matrix Theory (RMT) has produced broad insights in applied disciplines, with the universality of RMT particularly evident in quantum-resistant cryptosystems. In 1978, McEliece leveraged the NP-hardness of codes from random matrices in his yet-undefeated cryptosystem. Variants thereof have since

proliferated; in particular, Wangs 2017 RLCE system employs an encrypting block matrix constructed from a concatenation of random matrices. A key security assumption is the intractability of gleaning sensitive information about random matrices; however, RMT can unveil compromising traits of encrypting block matrices. Our goal is to prove the means by which the internal structure of random block matrices is betrayed by their limiting distribution.

We consider a variation of the RLCE matrix; let A and the sequence B_i be random square Hermitian matrices of dimension N and $2^{i-1}N$, respectively, drawn from ensembles with finite moments and different limiting distributions. We construct a "kaleidoscope" matrix by

$$K(A,B_i) = \begin{bmatrix} \begin{matrix} A & B_1 \\ B_1 & A \end{matrix} & B_2 & \multicolumn{2}{c}{\multirow{2}{*}{B_3}} & \cdots \\ B_2 & \begin{matrix} A & B_1 \\ B_1 & A \end{matrix} & & & \\ \multicolumn{2}{c}{\multirow{2}{*}{B_3}} & \begin{matrix} A & B_1 \\ B_1 & A \end{matrix} & B_2 & \\ & & B_2 & \begin{matrix} A & B_1 \\ B_1 & A \end{matrix} & \cdots \\ & \vdots & & \vdots & \ddots \end{bmatrix}.$$

We separate $K(A,B_i)$ into a sum of random matrices; the first is a diagonal block matrix D of A's, and the second is a hollow block matrix H of the sub-matrices B_i. For integer $k \geq 0$, we study

$$\mathbb{E}\left[\operatorname{Tr}\left(K(A,B_i)^k\right)\right] = \mathbb{E}\left[\operatorname{Tr}\left((D+H)^k\right)\right] = \sum_{s_1+t_1+\cdots+s_p+t_p=k} \mathbb{E}\left[\operatorname{Tr}\left(\prod_{j=1}^{p} D^{s_j} H^{t_j}\right)\right]$$

and prove that the limiting distributions of D and H are those of A and B_i, respectively. Resulting non-commutative matrix polynomials are the chief obstacle, but we re-interpret the trace operator as an inner product, allowing us to leverage analytic inequalities bounding the the non-commutative matrix products and prove the limiting distribution of $K(A,B_i)$ converges to the B_i ensemble. Computational results document rapid convergence even in cases of highly disparate component distributions, and we prove that the convergence rate is at least geometric. We develop sharp bounds for the minimum number of "kaleidoscopes" required to be within an arbitrary deviation of the B_i distribution, and show that finite iterations converge to new, universal distributions.

Zeros of L-Functions near the Central Point and Optimal Test Functions

Speakers: Charles Devlin
Supervisor: Steven Miller (Williams)

Abstract: Spacings between zeros of L-functions, most famously the Riemann zeta function, are central objects in modern number theory for their connection to many open problems, such as Chebyshev's bias and the class number problem. Montgomery and Dyson discovered in the 1970s that random matrix theory (RMT) seems to model the spacings between zeros of L-functions away from the central point $s = 1/2$. While we have an incomplete understanding as to why a correlation exists between RMT and number theory, this interplay has proved very useful for conjecturing answers to classical problems in number theory. Unfortunately these RMT models are insensitive to finitely many zeros, and thus completely miss the behavior near the central point. This is often the most arithmetically interesting place; for example, the Birch and Swinnerton Dyer conjecture states that the rank of the Mordell-Weil group equals the order of vanishing of

the associated L-function there. As these curves are of interest both in their own right and for applications such as elliptic curve cryptography, it is valuable to study the behavior near the central point.

Useful tools for this analysis are the n-level densities, introduced by Katz and Sarnak. They have many important connections with mathematical physics (their behavior is well modeled by RMT), and bound the average order of vanishing at the central point for a given family of L-functions by an integral of a weight against some test function ϕ (subject to support conditions of its Fourier transform, which prevent us from taking the Dirac delta functional and thus obtaining perfect knowledge at the central point). It is therefore of interest to choose ϕ optimally to minimize the integral and obtain the best bound possible. While the 1-level density has been studied in prior work, new technical problems emerge in the higher level densities which have left them relatively unexplored; however, larger values of n give better bounds for equivalent test function support, so there are compelling reasons to study the higher level densities. By restricting to a smaller class of test functions which split into products of test functions of a single variable, we are able to reduce the problem to an analogue of the one-dimensional case through a careful choice of one of the n test functions. This allows us to overcome the issues posed by the greater complexity of the higher level densities, as an appropriate choice yields a similar 1-level problem but with a different weighting function. We explicitly construct the space of admissible factors of the decomposition and prove, for a given set of $n-1$ admissible factors, the optimal choice of the final factor. Furthermore we compute the corresponding integral to give a formula for the value as a function of the factors in the decomposition of ϕ, obtaining strong estimates on both the average and excess ranks.

Thermodynamics of a Billiard-Simulated Gas

Speakers: Robin Armstrong
Supervisor: Yao Li (Umass)

Abstract: The derivation of macroscopic thermodynamic laws, such as Fouriers law, from microscopic classical mechanics is a fundamental challenge to physicists and mathematicians. In this study, we are interested in understanding thermodynamic properties of a microscopic heat conduction model that is based on dynamical billiards. Our motivation is that the thermodynamic properties of a gas, such as its temperature, entropy production, and thermal conductivity, arise essentially from the collisions of the gass constituent molecules, which is similar to the dynamics a collection of billiard balls colliding in an enclosed pool table. However, billiards are much more mathematically and computationally tractable than realistic gas molecules. Here, we present a computer algorithm which simulates a nonequilibrium billiards system that models microscopic heat conduction in gas. We wish to measure the thermodynamic properties of this simplified billiard-simulated gas, such as its heat flow and diffusion coefficient. We also wish to explore ways to increase the computational efficiency of this algorithm, and to investigate its relation to some simplified models that have been studied rigorously in earlier papers.

Symmetric variations of the Chan-Robbins-Yuen Polytope

Speakers: Max Liu (Mount Holyoke)
Supervisor: Alejandro Morales (UMass)

Abstract: Polytopes are convex objects of great combinatorial and geometric interest. The Chan-Robbins-Yuen (CRY) polytope is a polytope related to doubly-stochastic matrices and flows of graphs. One key feature of the CRY polytope is that its volume equals the products of consecutive Catalan numbers. This was proved algebraically by Zielberger in 1998: yet, there is no combinatorial proof known. We study two variations of this polytope by imposing symmetry conditions on the vertices or on the hyperplane description of the polytope. In this talk, I will discuss the number of vertices of these new polytopes and their volumes.

A Combinatorial Exploration of Chromino

Speakers: Nuha Futa (UMass)
Supervisor: Annie Raymond (UMass)

Abstract: We examined combinatorial questions in the Chromino game, which is a generalized chromatic version of the Domino game. We used Python coding to create a simulated two player game of Chromino. We used the information from several games to estimate averages for number of rounds, tiles left in bag, and size of the board. We created an upper bound on the number of possible games and used induction to prove this number.

Deep Learning of Biological Neurons

Speakers: Long Le
Supervisor: Yao Li (UMass)

Abstract: Given a small population of biological neurons, we can use integrate-and-fire model to investigate the neuronal dynamics due to the combination of external current drive (like sensory input) and the interaction among neurons of this population. It is known that some parameters of the model, such as the synapse connection strength and the synapse delay time, would significantly influence its dynamics. However, the explicit dependence between parameters and spiking pattern is unknown. In this project, we aim to use deep learning techniques to discover the hidden relation between spiking pattern and parameters. We use simulation results from randomly selected parameter sets as the input and those parameters as the output. The goal is to train an artificial neural network such that it can recover parameters after seeing the spiking pattern of a biological neuronal network.

Applying a Fokker-Planck Correction to the Perturbed Duffing Oscilator

Speakers: Mridul Madan
Supervisor: Matthew Dobson (UMass)

Abstract: Many models in physics and mathematical finance involve random processes which are governed by Stochastic Differential Equations (SDE). The Fokker-Planck equation describes the evolution of the probability density of the solution. Explicit solutions for most SDE's are unknown so numerical methods are used. Using a method proposed by Yao Li, we apply an improvement to a given numerical solution of a perturbed duffing oscillator for the case of a figure 8 orbit. This method improves and smooths the solution.

A Version of the Elephant Random Walk and Additive Functionals of Finite State Markov Chains

Speakers: Jonah Green, Taylor Meredith and Rachel Tan
Supervisor: Iddo Ben-Ari and Hugo Panzo (UConn)

Abstract: In this talk, we discuss a version of the Elephant Random Walk (Schütz and Trimper, 2004) with finite memory. Specifically, in our model, a walker carries a bag with a fixed number L of + and signs. At each time unit, the walker samples a sign from the bag at random. He then accepts it with probability p or flips it to the opposite sign with probability 1p, walks one step in the direction of the resulting sign, and returns that sign to the bag. We study the asymptotic behavior of the model through applications of classical results for additive functionals of Markov Chains. As a part of this work, we present an accessible derivation for the central limit theorem for additive functionals, focusing on the variance calculation.

The Voter Model on Complete Bipartite Graphs

Speakers: Philip Speegle, R Oliver Vandenberg
Supervisor: Iddo Ben-Ari and Hugo Panzo (UConn)

Abstract: The discrete-time Voter Model is a Markov chain on two-colorings of finite connected graphs, where at each time unit, a vertex is selected uniformly at random, adopting the color of a uniformly selected neighbor. Given enough time, the model will reach a ”consensus”, an absorbing state where all vertices are the same color. Conditioning on not reaching a consensus, the distribution of the process will approach a quasi-stationary distribution (QSD). In this work we study the QSDs for the model on complete bipartite graphs. For complete bipartite graphs where one group stays fixed in size and the other goes to infinity, we observed convergence to a distribution based on disagreements from the smaller group such that the proportion of colors approaches consensus.

On a Nonlinear Random Walk on Graphs

Speakers: Jonah Botvinick-Greenhouse, Connor Fitch, Mark Kong
Supervisor: Iddo Ben-Ari and Hugo Panzo (UConn)

Abstract: A simple random walk on a graph is a discrete-time dynamical system on probability distributions over the vertices in which the update function is linear. In a recent paper, Skardal and Adhikari propose a nonlinear update function in which the mass (probability) flow from vertex i to a neighboring vertex j is proportional to an exponential function of the mass at j. Our work further explores this model, giving methods to classify certain dynamics that arise in these systems, such as the stability of the uniform distribution as a fixed point for regular graphs and stationary distributions taking two values on complete graphs.