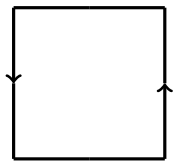
Honors Discovery Seminar: Introduction to Topology

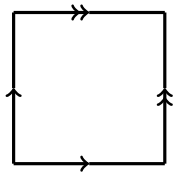
1 Part 1: Presentations

Imagine the following drawings are made out of an infinitely stretchy piece of clay. The different arrows and sides tell you how to glue corresponding edges together. See if you can determine what surface each of these presentations represents!

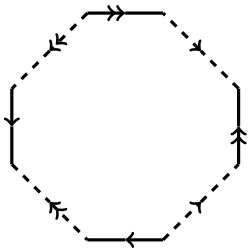
1. What surface is this?



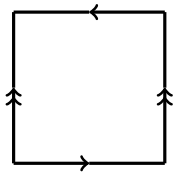
2. What surface is this?



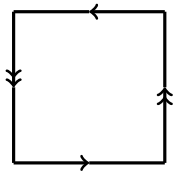
3. What surface is this?



4. What surface is this?



5. What surface is this?



6. Now that you've identified these surfaces, can you determine if any two of them are the same? How could you show that some of them are different from each other? Note: in topology, we say two surfaces are the same if one can be "smoothly deformed" into another. That's a fancy way of saying that we can squish our infinitely stretchy clay molded into one shape into another, without poking holes or introducing sharp corners.

2 The Mobius Band

1. Take a strip of paper, put one twist in it, and tape the sides together. This is called a mobius band. You may want to make more than one.

   (a) How many sides does this surface have? How many edges?

   (b) Is this surface the same as a cylinder? (Meaning, can it be smoothly deformed into a cylinder?) Why or why not? Your answer to (a) might help you here.

   (c) Draw a presentation of this surface, and use your presentation to predict what will happen when you cut your surface down the middle (the 'long' way).

   (d) Now, actually cut your surface down the middle. Is this what you expected? Explain what happened!

   (e) *Challenge:* From the presentation, what surface should you actually get? Can the surface you actually got be transformed into the one you predicted?

2. Use the presentation of the mobius band to figure out what will happen when instead of cutting it down the middle, you cut it down a third of the way from the top. Now make a mobius band, and cut it a third of the way down from the top. Explain what should happened, and then actually cut your surface.

3. When you have a surface with one edge, like a mobius band, the edge looks like a string with its ends joined together. Draw just the edge for a mobius band with one twist and a mobius band with three twists. Can you transform one into the other without tearing the string or passing it through itself? (This is a sub-field of topology called *knot theory*.)