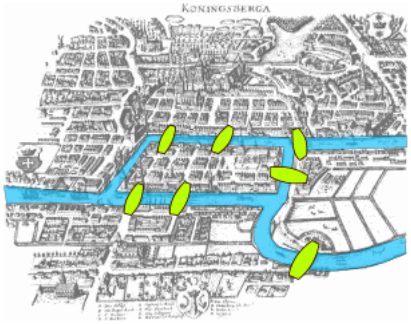
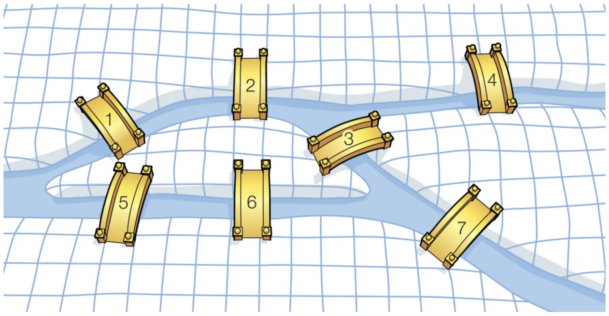
Honors Discovery Seminar: Introduction to Graph Theory

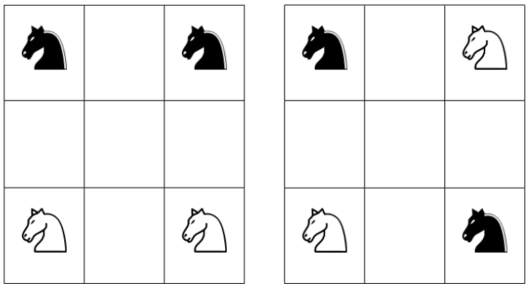
1. The city of Konigsberg has seven bridges. Is it possible to walk around the city, crossing every bridge exactly one? Below is a map of the city with the bridges highlighted, and a second more schematic map in case the first prints poorly.



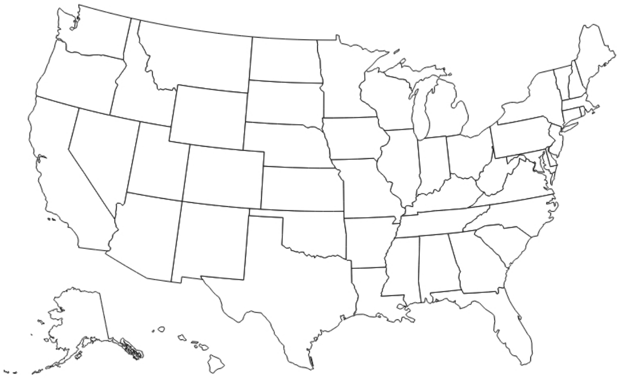


2. There are three houses in a new development, and they need to be connected to three utilities: water, sewer, and gas. If each house needs to connect to each utility, is it possible to do so without the pipes overlapping?

3. If you start with four knights placed on a 3×3 chess board as shown on the left, is it possible to move them into the position on the right if two knights are never allowed to occupy the same square?



4. Prove that it is not possible to color the US map below with three different colors so that the color of each state is not the same as the color of any state bordering it. (Is it possible with four colors?)



1. For each of the problems above, re-interpret it as a graph theory problem.

2. Brain-storm:
   (a) How would you draw a graph of a social network?
   (b) How would you draw a graph of the internet?
   (c) How could you represent a map and traffic flow with a graph?
   (d) How could you draw a graph to optimize air travel, in terms of minimizing the number of crew needed to operate every flight successfully?

3. Some applications of graph theory, which we will discuss together after finishing the other problems:
   (a) Targeted advertising (getting advertisements to desired groups of people)
   (b) Google's PageRank algorithm (how websites get ranked in order of important when you google something)
   (c) Any map/directions app (telling you what the fastest route is, using real-time data about traffic, road condisitons, etc)
   (d) Optimizing any sort of network flow (like air travel)
   (e) Classifying all possible platonic solids (three-dimensional shapes whose faces are identical regular polygons, with the same number of polygons meeting at each vertex)