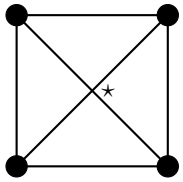
Honors Discovery Seminar: First Day Auction

You will work in small groups on the following problems for 30 minutes. At the end of the 30 minute time frame, teams will have the opportunity to auction off their solution: you will bid to present your answer, and the team with the highest bid wins the opportunity to present. After the presentation, if anyone thinks they have a better answer, the auction begins again. The team with the best answer wins! **No calculators, computers, phones, or checking the internet are allowed.**

1. Draw 7 dots on a piece of paper. Draw a line or curve segment connecting every pair of dots such that the curves cross each other as *few* times as possible, and no three curves can cross each other at the same point.

   For example, if we only had 4 dots, here is a drawing with one crossing (the crossing is labeled with a star):

   Your answer must be a finished drawing of the dots and lines/curves, with all crossings labeled.

2. Given any positive integer (whole number) n, create a sequence of numbers as follows: if n is even, divide it by 2. If n is odd, multiply it by 3 and add 1. Repeat on the output.

   For example, if we start with $n = 5$, the sequence is:

   $$5, 16, 8, 4, 2, 1.$$

   Find an integer n such that this process takes as *long* as possible to reach 1.

   Your answer must be given as the number n and its corresponding sequence.

3. Find the *largest* Fibonacci number that is also a prime number.

   As a reminder, the Fibonacci numbers are defined by the relationship $F_n = F_{n-1} + F_{n-2}$ (take the previous two numbers and add them together), so the series starts as:

   $$1, 1, 2, 3, 5, 8, 13, 21, \ldots$$

4. A unit-distance coloring of the xy-plane is a way to color the plane such that any two points exactly 1 unit apart are colored with different colors. Find a unit-distance coloring of the plane using as *few* colors as possible.

   For example, here is a coloring of the plane with 16 colors (each number represents a different color; the side length of each square is $1/\sqrt{2}$; the boundary grid lines are considered as the color to the right or above, and the boundary corners are considered to be the color to the above right; and the pattern repeats indefinitely).

1	2	3	4	1
13	14	15	16	13
9	10	11	12	9
5	6	7	8	5
1	2	3	4	1

   Your answer must be given a drawing of part of the plane that repeats indefinitely, with the number of colors labeled.