Honors Discovery Seminar: Combinatorics

Combinatorics is the math of **counting things**. Here are some examples of things we can count.

1. How many ways are there to rearrange three items, numbered one, two, three? What about four items? What about n items?

2. How many ways are there to choose a group of three people from a class of 8 people? What about choosing a group of 4? What about k people from a group of n?

3. If you flip a coin 8 times, how many ways are there to get three heads? What about four heads? How many ways are there to get k heads if you flip a coin n times?

4. The usual configuration of bowling pins in four rows, with four pins in the back, three in the next row, two in the next, and one in the front. How many pins is this? How many pins would we need for five rows? Six rows? n rows?

5. A *partition* of a positive integer n is a sum of non-increasing positive numbers equalling n. For example, if $n = 2$, the partitions are:
$$2 = 2 \quad \text{and} \quad 2 = 1 + 1$$
and if $n = 3$, the partitions are:
$$3 = 3 \quad \text{and} \quad 3 = 2 + 1 \quad \text{and} \quad 3 = 1 + 1 + 1.$$
How many partitions are there of 4? What about n?

6. How many ways are there to put 2 balls into 4 bins? What about about 3 balls? 4 balls? 5 balls? n balls?

7. How many zig-zag paths are there from the point $(0, 0)$ to the point $(3, 3)$, if we can only take steps one unit to the right or one unit up? How many paths are there if we can't go above the line $y = x$? What if we need to get to the point (n, n)?