

# Practice problems from old exams for math 132

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**Disclaimer:** Your instructor covers far more materials that we can possibly fit into a four/five questions exams. These practice tests are meant to give you an idea of the kind and varieties of questions that were asked within the time limit of that particular tests. In addition, the scope, length and format of these old exams might change from year to year. Users beware! These are NOT templates upon which future exams are based, so don't expect your exam to contain problems exactly like the ones presented here.

## 1 Practice problems for Exam 1.

1. Evaluate the following integrals algebraically. Show all your steps.

(a)  $\int \frac{(\ln x)^3}{x} dx$

(b)  $\int_1^8 (x^{\frac{2}{3}} - \frac{4}{x}) \sqrt[3]{x} dx$

(c)  $\int_0^3 (\sqrt{1+x})x dx$

2. Set up a definite integral for the volume of the solid obtained by rotating about the  $x$ -axis the region bounded by  $y = \sqrt{25 - x^2}$  and  $y = 3$ . Do *NOT* evaluate the integral.

3. Sketch the region bounded by the curves  $y = \cos(x)$ ,  $y = \sin(x)$ ,  $x = -\frac{\pi}{4}$ , and evaluate its area.

4. Suppose  $f(x)$  is a continuous function on the interval  $[-4, 4]$  and  $\int_{-4}^0 f(x) dx = -7$ ,  $\int_{-4}^4 f(x) dx = -7$ ,  $\int_0^1 f(x) dx = 1$ .

Evaluate, showing all your steps,

(a)  $\int_1^4 f(x) dx$

(b)  $\int_0^2 f(x^2)x dx$

5. If  $F(x) = \int_1^{\ln(x)} e^{t^2} dt$

(a) Find  $F'(x)$ .

(b) Find  $F(e)$ . Justify your answers!

6. (a) Approximate the area under the curve  $y = x^2$  from 2 to 3

i. Using a Riemann sum with 4 intervals and *left* endpoints.

- ii. Using a Riemann sum with 4 intervals and *right* endpoints.
- (b) Which of the two is a better approximation? Justify your answer!
7. A particle moves along the  $x$ -axis in a straight line with velocity  $v(t) = t^2 - t - 6$  for  $1 \leq t \leq 4$  (measured in ft/sec).
- (a) Determine the **total displacement** of the particle between the times  $t = 1$  and  $t = 4$ .
- (b) Determine the **total distance** that the particle traveled between  $t = 1$  and  $t = 4$ .
8. (a) Consider the function  $f(x) = 16 - x^2$ . Approximate the area under this curve between  $x = 0$  and  $x = 4$ , using
- four rectangles and left endpoints;
  - four rectangles and right endpoints;
  - Can you state whether each of the above answers constitutes an over-estimate, or an under-estimate of the area and briefly explain why?
- (b) Evaluate the definite integral

$$\int_{-2}^2 \sqrt{4 - x^2} dx$$

by interpreting it in terms of areas.

9. (a) Find a function  $f$  such that
- $f''(x) = 6x + 12x^2$
  - $f(1) = 5$ ,  $f'(1) = -3$ .
- (b) If  $\int_0^5 f(x) dx = 30$ ,  $\int_5^8 f(x) dx = 7$ ,  $\int_0^2 g(x) dx = 9$ , and  $\int_2^8 g(x) dx = 7$ , find  $\int_0^8 (2f(x) + 3g(x)) dx$ .
10. (a) For the function:

$$F(x) = \int_{\cos x}^0 \frac{t^2 + 1}{e^t} dt,$$

- find  $F'(x)$
  - find  $F(\frac{\pi}{2})$ .
- (b) Evaluate the general indefinite integral:

$$\int (\sin(x) + x\sqrt{x} + x^{80} + 7e^{2x}) dx$$

11. (a) Evaluate the general indefinite integral

$$\int \frac{1}{x \ln^3(x)} dx$$

- (b) Compute, showing all your work, the definite integral

$$\int_0^1 \frac{3 + 4x}{1 + x^2} dx$$

12. (a) Find the area enclosed between the curves  $y = 4 - x^2$  and  $y = x^2 - 4$ , by sketching it, then setting up and evaluating an appropriate definite integral.  
(b) Find the volume of the solid obtained by rotating the region between the curves  $y = x$  and  $y = x^4$ , around the  $y = -1$  line.
13. Use appropriate substitutions to evaluate the following integrals. For each, clearly indicate the substitution you are using.

(a)  $\int_1^3 x e^{x^2} dx$

(b)  $\int \tan^3 r \sec^2 r dr$

(c)  $\int \frac{t^5}{\sqrt{t^2+3}}$

14. (a) Find the exact value of

$$\int_1^3 (1 + 2x) dx$$

*geometrically* by interpreting it as an area.

- (b) Approximate the same integral  $\int_1^3 (1 + 2x) dx$  by a Riemann sum that uses **4** equal-length subintervals and **right**-hand endpoints as the sample points. (Show the individual terms of the Riemann sum before you calculate the value of the sum.)

15. An object is moving along a line, with velocity

$$v(t) = t^2 - 4t + 3 \text{ feet/second,}$$

until it comes to a stop at time  $t = 3$  seconds. Use definite integrals to answer the following questions:

- (a) How far away is the object at time  $t = 3$  from where it was at time  $t = 0$ ?  
(b) How many feet did the object travel since time  $t = 0$  until time  $t = 3$ ?

16. Starting at time  $t = 0$  minutes, a metal bar is cooling at the rate

$$r(t) = 98e^{-0.1t} \text{ }^\circ\text{C}/\text{min}.$$

How much will the temperature of the bar have changed by the end of an hour?  
(Use a definite integral to obtain your answer.)

17. Express each of the following as a definite integral. However, **do not evaluate that integral.**

- (a) The area of the region enclosed by the graphs of  $y = 1 - 3x$  and  $y = 1 - x^2$ .  
(b) The volume of the solid obtained by rotating that region around the  $y$ -axis.

18. Determine the following derivatives. Briefly justify each answer!

- (a)  $\frac{d}{dt} \left( \int \sin(\sqrt[3]{e^t + t^{132}}) dt \right)$   
(b)  $\frac{d}{dt} \left( \int_t^\pi \sin(\sqrt[3]{e^x + x^{132}}) dx \right)$   
(c)  $\frac{d}{dt} \left( \int_\pi^{2005} \sin(\sqrt[3]{e^t + t^{132}}) dt \right)$

19. The continuous functions  $f$  and  $g$  have the properties:

$$\int_5^8 3g(x) dx = 12, \quad \int_5^8 [2f(x) + g(x)] dx = 12, \quad \int_0^5 f(x) dx = 3.$$

- (a) Find the value of  $\int_0^8 f(x) dx$ .  
(b) If also

$$\int_0^{10} f(x) dx = 15,$$

then find the value of  $\int_0^5 f(2x) dx$ .

20. Let  $f(x) = x^2 + 10$ .

- (a) Approximate the definite integral  $\int_2^{10} f(x) dx$  by the Riemann sum that uses **4** subintervals and **right** endpoints as the sample points. Show the individual terms of the sum before you calculate the value of the sum.  
(b) Draw a picture of the graph of  $f(x)$ . On the graph, draw the rectangles that correspond to the Riemann sum in (a).

21. Water is being pumped into a storage tank at a rate of

$$r(t) = 60e^{-\frac{t}{15}} \text{ liters per second}$$

where time  $t$  is measured in seconds. How much water is pumped into the tank during the entire time interval  $30 \leq t \leq 40$ ? Use a definite integral to arrive at your answer.

22. Sketch the region enclosed by the curves  $y = x$  and  $y^2 = x + 2$  and then determine its area.
23. Use appropriate substitutions to evaluate the following integrals. For each, clearly indicate the substitution you are using. Show all your work. (You may use your calculator to check your answer.)

(a)  $\int \frac{t^2}{\sqrt{t-1}} dt$

(b)  $\int_0^1 \frac{\arctan x}{x^2+1} dx$  Note:  $\arctan$  is the inverse function of  $\tan x$ .)

24. A particle moves along the  $x$ -axis with velocity  $v(t) = t^3 - 3t$  for  $0 \leq t \leq 5$ . The particle is at the origin at time 0.

- (a) What is the **total distance** that the particle traverses - the "odometer distance" - between times  $t = 0$  and  $t = 5$ ?
- (b) What is the  $x$ -coordinate of the particle at time  $t = 5$ ?

25. The plane region in the first quadrant that is enclosed by the curves  $y = 0$ ,  $y = \sin x$ , and  $x = \pi$  is rotated around the line  $y = 1$  to form a solid.

- (a) Find a formula for the area  $A(x)$  of cross-sections of the solid formed by planes perpendicular to the axis of rotation.
- (b) Express the volume of the solid as a definite integral. **Do NOT evaluate that integral!**

26. Find the following and **justify your answers**.

- (a) The value of

$$\int_0^1 \frac{d}{dx} \left( \frac{e^{x^2+1}}{x^2+4} \right) dx.$$

- (b) For  $x > 0$ , the **derivative** of the function

$$F(x) = \int_e^{\sqrt{x}} \ln t dt.$$

27. (a) Find the general indefinite integral:

$$\int \frac{\ln(x)}{x} dx$$

- (b) Evaluate the integral:

$$\int_1^3 \sqrt{x} \left( x^{\frac{3}{2}} - \frac{3}{x} \right) dx$$

28. An animal population is increasing at a rate given by the function  $f(t) = 10e^{2t}$  animal/year. What is the total change in animal population between the fifth ( $t = 5$ ) and the seventh ( $t = 7$ ) year?

29. If

$$F(x) = \int_0^{\sin x} te^t dt,$$

- (a) find  $F'(x)$   
(b) find  $F(\pi)$ .
30. Estimate the area under the graph of  $f(x) = x^2 + 2$ , between 0 and 4,  
(a) Using 4 rectangles and **left** endpoints. Plot the relevant graph showing the rectangles.  
(b) Is your estimate in (a) and underestimate or an overestimate?  
(c) Find the actual area.
31. The velocity of a particle is given by  $v(t) = 6 - 3t$  m/sec. What is the *total displacement* of the particle between  $t = 0$  sec and  $t = 3$  sec? What is the *total distance* it has traveled between these times?
32. A particle (that can move along the  $x$ -axis) at  $t = 0$  sec is at  $x = 0$  m, with speed  $v = 0$  m/sec. From that moment onwards the particle has a constant acceleration  $a = 5$  m/sec<sup>2</sup>. At what distance  $x$  (in meters) will the particle have a speed of  $v = 10$  m/sec?
33. Sketch the region enclosed by the curves  $y = |x|$  and  $y = x^2 - 2$  and evaluate its area.
34. Find the volume of the solid obtained by rotating the region bounded by  $y = \frac{1}{\sqrt{x}}$  and  $y = x$ , between  $x = 1$  and  $x = 2$  around the  $x$ -axis.

## 2 Practice problems for "Integral" Exam 2.

Aside from the most basic relations such as  $\tan x = \frac{\sin(\theta)}{\cos(\theta)}$  and  $\sec x = \frac{1}{\cos(\theta)}$ , you should know the following trig identities:

$$\cos^2(\theta) + \sin^2(\theta) = 1.$$

$$\sec^2(\theta) - \tan^2(\theta) = 1.$$

$$\cos^2(\theta) = \frac{1 + \cos(2\theta)}{2}$$

$$\sin^2(\theta) = \frac{1 - \cos(2\theta)}{2}$$

$$\sin(2\theta) = 2 \sin(\theta) \cos(\theta)$$

$$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta) = 2 \cos^2(\theta) - 1 = 1 - 2 \sin^2(\theta)$$

Find the general antiderivative of each of the following. You must show the details of techniques that you use. **No calculator!**

1.  $\int x \cos(x^2 + 2) \sin(x^2 + 2) dx$

2.  $\int \frac{\cos x}{1 + \sin x} dx$

3.  $\int \sqrt{t} \ln t dt$

4.  $\int x e^{x^2} dx$

5.  $\int \sin^2 x dx$

6.  $\int \cos^4 x \sin x dx$

7.  $\int \tan x \sec x dx$

8.  $\int \ln(x^2) dx$

9.  $\int x \cos^2 x \, dx$

10.  $\int \tan x \sec^2 x \, dx$

11.  $\int (e^{-5x} + \sin(3x)) \, dx$

12.  $\int \left( 2e^x + \frac{3}{\sqrt{1-x^2}} + \sec(x) \tan(x) \right) dx$

13.  $\int \frac{\sqrt{x} + x^2 + 1}{x} \, dx$

14.  $\int \sin^2(x) \cos^3(x) \, dx$

15.  $\int x^4 \ln(x) \, dx$

16.  $\int \frac{(\sqrt{x} + 4)^5}{\sqrt{x}} \, dx$

17.  $\int x^2 e^x \, dx$

18.  $\int x^3 \sqrt{x^2 + 1} \, dx$

19.  $\int x (\sin^2(x^2)) \, dx$

20.  $\int \frac{x + \sqrt{x}}{x^2} + \frac{3}{x^2 + 1} \, dx$

21.  $\int \frac{e^{\sqrt{x}}}{\sqrt{x}} \, dx$

22.  $\int \cos^3(\theta) \sin^4(\theta) \, d\theta$

23.  $\int x \ln(x) \, dx$

24.  $\int x^2 \cos(2x) \, dx$



25.  $\int \sqrt{9 - x^2} dx$

26.  $\int \cos(x) \sin(x) e^{\sin(x)} dx$

27.  $\int (4x^5 + 3e^{4x} - \sqrt{x} + 7 \sec^2(x) - 33) dx$

28.  $\int \tan^2(x) \sec^2(x) dx$

29.  $\int \frac{e^x}{5 + e^x} dx$

30.  $\int \sqrt{x} \ln(x) dx$

31.  $\int x \sec^2(x) dx$

32.  $\int \sin^2(x) \cos^3(x) dx$

33.  $\int \sin^2(x) \cos^2(x) dx$

34.  $\int x^3 \sqrt{x^2 + 1} dx$

35.  $\int x^5 \sqrt{25 - x^2} dx$

36.  $\int (\cos x + 5 \sin x) dx$

37.  $\int \left( \sqrt{x} + \frac{1}{\sqrt{x}} \right)^2 dx$

38.  $\int \sqrt{2x + 4} dx$

39.  $\int x e^{-2x} dx$

40.  $\int \frac{\sin x}{1 + \cos^2 x} dx$

41. 
$$\int \frac{x+3}{x^2-3x+2} dx$$

42. 
$$\int x^2 \sin x dx$$

43. 
$$\int \frac{2x-1}{\sqrt{4-x^2}} dx$$

44. 
$$\int \tan^3 x \sec^5 x dx$$

45. 
$$\int \frac{e^{2x}}{e^x-2} dx$$

46. 
$$\int (x\sqrt{x} + \sin 2x) dx$$

47. 
$$\int \frac{\cos(\ln x)}{x} dx$$

48. 
$$\int x^2 \ln x dx$$

49. 
$$\int x^2 \sqrt{x+1} dx$$

50. 
$$\int \frac{x^3-4x-10}{x^2-x-6} dx$$

51. 
$$\int \frac{\cos x}{1+\sin x} dx$$

52. 
$$\int x^2 \cos x dx$$

53. 
$$\int \frac{x^2}{\sqrt{4-x^2}} dx$$

54. 
$$\int \frac{\tan^3 x}{\cos^3 x} dx$$

55. 
$$\int \frac{1}{1+e^{-x}} dx$$

### 3 Practice problems for Exam 3.

1. Does the series converge? Why or why not? (Name any test you use and show that the conditions needed for that test are satisfied.)

(a)  $\frac{1}{5} - \frac{3}{25} + \frac{9}{125} - \frac{27}{625} + \dots$

(b)  $\sum_{n=1}^{\infty} \frac{3n^7}{8n^7 + 6n^2 - 5n - 2}$

(c)  $\sum_{n=1}^{\infty} \frac{\sqrt{n^3 + 2n}}{n^4 - 7}$

(d)  $\sum_{n=1}^{\infty} (-1)^n \frac{n^3 - n}{4n^5 + n^3 + 4}$

2. Find the interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(x-1)^n}{(n+1)3^n}$$

3. Find a power series representation for  $f(x) = \frac{4}{1+2x}$  around  $a = 0$ ; write the power series using sigma ( $\Sigma$ ) notation. State for which  $x$  the power series actually has sum  $f(x)$ .
4. The sequence  $\{S_n\}_{n=1}^{\infty}$  of partial sums of the series  $\sum_{n=1}^{\infty} a_n$  is given by

$$S_n = \frac{n}{5+n} \quad \text{for } n = 1, 2, 3, \dots$$

- (a) Does the series  $\sum_{n=1}^{\infty} a_n$  converge? If so, to what sum?
- (b) Use the formula for  $S_n$  to find the values of  $a_1$  and  $a_2$ . Then find an explicit formula in for  $a_n$  in terms of  $n$ .
5. Consider the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(n+1)!} x^n.$$

- (a) Determine the radius of convergence of this power series.
- (b) Show that this power series converges for  $x = \frac{1}{3}$ .
- (c) How many terms do you need to use to approximate the infinite series in (b) so that the error is less than 0.001?
6. Find the first three terms of the Taylor series for the function  $f(x) = x \ln(x^2 + 1)$  centered around  $a = 1$ .

7. (a) Starting with the MacLaurin series for  $e^x$  (which you may just write down), obtain a power series representation of  $e^{-x^2}$ .
- (b) Use your answer to (a) to express  $\int_0^1 e^{-x^2} dx$  as the sum of an infinite series.
- (c) Find a bound on the error if the first three terms of the series you obtained in (b) were used to approximate  $\int_0^1 e^{-x^2} dx$ . (Do *not* actually calculate the approximation and do *not* use any value for this integral that your calculator might give.)

8. Evaluate the definite integral

$$\int_0^1 \frac{x^3}{\sqrt{3+x^2}} dx$$

9. Evaluate the indefinite integral

$$\int x^2 e^{3x} dx$$

10. Evaluate the definite integral

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sin^5 x \cos^3 x dx$$

11. Examine whether the improper integral

$$\int_1^{\infty} \frac{\ln(x)}{x^3} dx$$

converges or diverges. If it converges, evaluate it.

12. Examine whether the improper integral

$$\int_3^{\infty} \frac{1}{x-2} dx$$

converges or diverges. If it converges, evaluate it.

13. Find the exact value of the repeating decimal  $0.\overline{15} = 0.151515\dots$  as a rational number.

14. Does the series converge? Why or why not? (Name any test you use and show that the conditions needed for that test actually hold.)

(a)  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}$

(b)  $\sum_{n=1}^{\infty} \frac{3^n}{10^n}$

- (c)  $\sum_{n=1}^{\infty} \frac{n^2+1}{7n^2+3n-5}$
- (d)  $\sum_{n=1}^{\infty} \frac{\sqrt{n^4+1}}{n^4+3n+5}$
15. (a) Approximate the sum of the series  $\sum_{n=1}^{\infty} \frac{1}{n^5}$  by the sum of its first **three** terms. Round your answer to six decimal places.
- (b) Without using any value for the actual sum of the series, find an upper bound for the error involved in that approximation. Round your answer to **six** decimal places.
16. Find the radius of convergence and the interval of convergence of the power series  $\sum_{n=1}^{\infty} \frac{(x-3)^n}{n \cdot 2^n}$ .
17. Find the **first 4 terms** in the Taylor series for  $f(x) = \sin x$  about  $\frac{\pi}{4}$ .
18. (a) Find a power series representation for the function  $\frac{1}{1+x^4}$ . Use summation  $\Sigma$ -notation to express your answer.
- (b) Use (a) to express  $\int_0^{\frac{1}{2}} \frac{1}{1+x^4} dx$  as the sum of an infinite series.
19. Using the root test, find whether the series is convergent or divergent:

$$\sum_{n=1}^{\infty} \left( \frac{n^2 + 2}{2n^2 - 1} \right)^n$$

20. Determine whether the improper integral converges and, if so, find its value:

- (a)  $\int_{-\infty}^1 x e^{-x^2} dx$
- (b)  $\int_1^{\infty} \frac{1}{\sqrt{x}(1+x)} dx$
- (c)  $\int_1^2 \frac{1}{(x-1)^2} dx$
- (d)  $\int_0^2 \frac{x}{\sqrt{4-x^2}} dx$

21. For each of the following two methods, set up a sum that, if you did the arithmetic, would approximate  $\int_0^1 \frac{1}{x^2+1} dx$ :
- (a) The trapezoidal rule with  $n = 6$  subintervals.
- (b) Simpson's rule with  $n = 6$  subintervals.

22. Determine whether the sequence  $\{a_n\}_{n=1}^{\infty}$  converges and, if so, find its limit:

(a)  $a_n = \frac{3n^2 + 4n}{5 + 6n^2}$

(b)  $a_n = \frac{\sin n}{n^2}$

(c)  $a_n = \frac{(1) + (-1)^n}{n + 2}$

(d)  $a_n = \frac{4^{n+1}}{7^{2n}}$

#### 4 Practice problems for Final Exam.

1. Determine whether each of the statements below is true or false. If a statement is false, briefly explain *why* it is false.

(a) The most general antiderivative of  $x^2 + \sin(x)$  is  $\frac{x^3}{3} - \cos(x)$ .

(b)  $\int_a^b f(x) dx = - \int_b^a f(x) dx$ .

(c) If  $f$  is continuous on  $[a, b]$ , then  $\int_a^b f(x) dx = F(b) - F(a)$  where  $F' = f$ .

(d) If  $f$  is even, then  $\int_{-a}^a f(x) dx = 0$ .

2. Evaluate the indefinite integral, showing *all* the intermediate steps of the calculation:

$$\int \sqrt{1 + x^2} x^5 dx$$

3. Evaluate the indefinite integral, showing *all* the intermediate steps of the calculation:

$$\int x \cos(\pi x) dx$$

4. Examine whether the *improper* integral

$$\int_0^1 \frac{1}{4x - 1} dx.$$

is convergent or divergent. Evaluate it, if it is convergent. Show *all* the intermediate steps of the calculation.

5. Is the series

$$\sum_{n=2}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n^2-1}}$$

convergent? Is it absolutely convergent? You should clearly explain and justify your answer.

6. Is the following series convergent or divergent?

$$\sum_{n=1}^{\infty} \frac{\sqrt[3]{n^6+n^2+1}}{n^4-n+3}$$

You should clearly explain and justify your answer.

7. Use the integral test to verify that the following series is divergent:

$$\sum_{n=2}^{\infty} \frac{1}{n \log(n)}$$

8. Convert the decimal number
- $0.352352352352352\dots$
- into a fraction using the geometric series. Show all the details of the calculation.

9. Determine whether the series

$$\sum_{n=1}^{\infty} \frac{3^n}{(2n)!}$$

is convergent or divergent. You should clearly explain and justify your answer.

10. Determine the interval of
- $x$
- for which the series

$$\sum_{n=1}^{\infty} \frac{5^n(x-2)^n}{n}$$

is convergent. You should clearly explain and justify your answer.

11. Approximate
- $\int_0^2 \ln(1+x^3-x) dx$
- in two ways:

- (a) By the Riemann sum with  $n = 4$  subintervals and **right** endpoints.  
 (b) By using the **Trapezoidal Rule** with  $n = 4$ .

12. (a) Calculate the area of the region bounded by
- $x = |y|$
- and
- $x = y^2 - 2$
- .

- (b) Set up - but do **not** evaluate - an integral for the volume of the solid obtained by rotating around the horizontal line  $y = 2$  the region bounded by the curves  $y = 1$ ,  $y = x^3$ , and  $x = 0$ .

13. Use the relevant techniques of integration to evaluate:

(a)  $\int x \cos x \, dx$

(b)  $\int \frac{x+1}{x^2+5x+6} \, dx$

14. Does the series converge? Why or why not?

(a)  $\sum_{n=0}^{\infty} 2^{n+1} 3^{-n}$

(b)  $\sum_{n=3}^{\infty} \frac{1}{n(\ln n)^2}$

15. Given that  $f$  is a continuous function with

$$\int_0^2 f(x) \, dx = 3, \quad \int_0^5 f(x) \, dx = -3, \quad \int_1^5 f(x) \, dx = 9, \quad \int_1^2 f(x) \, dx = 15,$$

find the value of  $\int_0^2 x f(x^2 + 1) \, dx$ .

16. Find the derivative:

$$\frac{d}{dx} \left( \int_{-1}^{x^2+1} e^{\cos t} \, dt \right)$$

17. Find the **interval** of convergence for  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \frac{(x-1)^n}{2^n}$ .

18. Find the coefficient of  $x^4$  in the Taylor series for  $f(x) = \ln(\cos x)$  about  $x = 0$ .

19. Consider the curve having polar equation  $r = 2 \sin \theta$ .

(a) Write *parametric* equations for this curve using  $\theta$  as the parameter:

$$\begin{cases} x = \\ y = \end{cases}$$

(b) Use either the polar equation or your parametric equations to obtain an equation for the tangent line to this curve at the point where  $\theta = \frac{\pi}{2}$ .



- (c) Use either the polar equation or your parametric equations to express the length of this curve, for  $0 \leq \theta \leq 2\pi$ , as a definite integral. (You need not evaluate that integral.)

20. The Maclaurin series representation for  $\sin x$  is:

$$\sin x = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}$$

- (a) Use this series to find a power series for  $g(x) = \sin(x^2)$ . Either show at least six non-zero terms or else give your answer using  $\Sigma$ -notation.
- (b) Find a series that converges to  $\int_0^1 \sin(x^2) dx$ . Either show at least six non-zero terms or else give your answer using  $\Sigma$ -notation.
- (c) Obtain an upper bound on the error in approximating the exact value of  $\int_0^1 \sin(x^2) dx$  by the sum of the first **three** non-zero terms from (b). (Do this without actually finding a numerical value for either the sum of those three terms or the definite integral.)
21. Approximate  $\int_0^1 e^{-x^2} dx$  in each of the following ways:
- (a) By forming the Riemann sum with  $n = 4$  subintervals and **left** endpoints as sample points.
- (b) By using Simpson's Rule with  $n = 4$  subintervals.
22. (a) Find the area of the bounded region that is enclosed by the curves  $y = x^2 - 9$  and  $y = 9 - x^2$ .
- (b) Find the volume of the solid obtained by rotating around the  $x$ -axis the region bounded by the curve  $y = \tan x$  and the lines  $y = 0$  and  $x = \frac{\pi}{4}$ .

23. Does the infinite series  $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$  converge? Why or why not?

24. Find the **interval** of convergence of the power series  $\sum_{n=0}^{\infty} \frac{(x-1)^n}{(n+1) \cdot 3^n}$ .

25. The rate  $r$  at which people become ill with the flu at time  $t$  days after an epidemic begins is given by  $r = 1000te^{-\frac{t}{20}}$  people per day. How many people become ill with the flu during the first 10 days of this epidemic?

26. A rumor is spreading across campus. The time  $t(p)$ , in hours, by which the *fraction*  $p$  of all students have heard the rumor is given by

$$t(p) = \int_{0.01}^p \frac{4.8}{x(1-x)} dx.$$

How long does it take for 90% of all the students to have heard the rumor?

27. Find a power series representation for  $f(x) = \frac{4}{1+2x}$  around  $a = 0$ ; write the power series using sigma  $\Sigma$ -notation. State for which  $x$  the power series actually has sum  $f(x)$ .

28. Calculate the degree 3 Taylor polynomial  $T_3(x)$  of  $g(x) = \frac{1}{\sqrt{x}}$  around  $a = 1$ .

29. Find the values of:

(a)  $\int_0^1 \frac{d}{dt} (\sqrt{1+t^3}) dt$

(b)  $\frac{d}{dx} \left( \int_0^1 \sqrt{1+t^3} dt \right)$

(c)  $\frac{d}{dx} \left( \int_0^{x^2} \sqrt{1+t^3} dt \right)$

30. As the parameter  $t$  increases forever, starting at  $t = 0$ , the curve with parametric equations

$$\begin{cases} x = e^{-t} \cos t, \\ y = e^{-t} \sin t \end{cases}$$

spirals inward toward the origin, getting ever closer to the origin (but never actually reaching) as  $t \rightarrow \infty$ . Find the length of this spiral curve.

31. (a) Starting with the Maclaurin series for  $e^x$  (which you may just write down), obtain a power series representation of  $e^{-x^2}$ .

(b) Use your answer to (a) to express  $\int_0^1 e^{-x^2} dx$  as the sum of an infinite series.

- (c) Find a bound on the error if the first three (nonzero) terms of the series you obtained in (b) were used to approximate  $\int_0^1 e^{-x^2} dx/4$ . (Do *not* actually calculate the approximation and do *not* use any value for this integral that your calculator might give!)

32. Express the parametric curve

$$x = -3 + \cos t$$

$$y = 1 + \sin t$$

in Cartesian  $xy$ -coordinates. Also, compute its slope  $\frac{dy}{dx}$  at  $t = \frac{\pi}{3}$ .

33. Compute the length of the curve

$$x = e^t \cos t, \quad y = e^t \sin t, \quad 0 \leq t \leq \pi.$$

34. (a) Consider the curve given in polar coordinates by the expression

$$r = 2 + \cos \theta$$

- i. Sketch this curve.
  - ii. Find the area enclosed by the curve.
- (b) Find the vertical tangents to the curve

$$r = 2 + \cos \theta$$

[find both the angle  $\theta$  at which they occur and the actual equation of the vertical tangent]. To do this, find the slope  $\frac{dy}{dx}$  of the polar curve and identify for which  $\theta$ 's the slope becomes infinite.

35. Find the area of the region bounded
- $y = -x^2 + 8x - 11$
- and
- $y = 2x - 6$
- .

36. Find
- $k$
- so that

$$1 = \int_0^{\infty} k e^{-4t} dt$$

37. Let
- $C$
- be the curve described by
- $x(t) = 2 + 2 \cos t$
- and
- $y(t) = 4 + 4 \sin^2 t$
- for
- $0 \leq t \leq \pi$
- .

- (a) Find  $(\frac{dy}{dx})$ .
- (b) Find the  $x$  and  $y$  of the point where  $C$  has a horizontal tangent line.
- (c) Find an equation of the line tangent to  $C$  at  $(3, 7)$ .

38. Let
- $C$
- be the curve described by
- $x(t) = 1 - 2 \cos^2 t$
- and
- $y = 3 \sin^2 t$
- for
- $0 \leq t \leq \pi$
- .

- (a) Draw a graph of  $C$  with a window:  $-2 \leq x \leq 4$  and  $-1 \leq y \leq 4$ .
- (b) Eliminate the parameter  $t$ . That is, find an equation that describes  $C$  with only variables  $x$  and  $y$ .
- (c) A particle travels along  $C$  from time 0 to time  $\pi$ . Find the distance that particle travels, i.e. the arc length of  $C$  from  $t = 0$  to  $t = \pi$ .

- (d) Find the distance from the initial point  $(x(0), y(0))$  to the terminal point  $((x(\pi), y(\pi)))$ , i.e. the displacement of the particle.
39. Find the area inside of  $r = 2$  and to the right of  $r = \sec \theta$ .
40. Find a power series that represents the function:
- (a)  $\frac{2x}{1+x^2}$   
 (b)  $\ln(1+x^2)$ .
- [Hint:  $\sum_{n=0}^{\infty} (-1)^n x^{2n} = \frac{1}{1+x^2}$ ]
41. Write down the MacLaurin series for
- (a)  $e^x$   
 (b)  $\cos x$
42. Let  $F(x) = \int_0^x \sin(t^3) dt$ . Find the Maclaurin series of  $\frac{d(F(x))}{dx}$ .
- [Hint:  $\sin z = \sum_{k=0}^{\infty} (-1)^k \frac{z^{2k+1}}{(2k+1)!}$ ]
43. Suppose that  $f^{(n)}(0) = \left(\frac{2}{3}\right)^n (n!)$ .
- (a) Find the Maclaurin series of  $f(x)$ .  
 (b) Find the radius of convergence.
44. (a) Using the Taylor's polynomial (with  $a = 0$ ) for the  $\sin x$  with a 5<sup>th</sup> degree term compute an approximation of  $\sin 2$ .  
 (b) Using Taylor's inequality find an upper bound for your answer in (a).
45. A particle moves on a straight line with initial velocity 4 m/s and acceleration given by  $a(t) = -1 \text{ m/s}^2$ .
- (a) Find the total distance traveled by the particle during the first 6 seconds.  
 (b) Find the total displacement of the particle during the first 6 seconds.
46. (a) Sketch the area enclosed by the curves  $y = x^2 - 4$  and  $y = 4 - x^2$ .  
 (b) Evaluate this area.
47. Evaluate the integral (show your work):

$$\int_0^{\frac{\pi}{2}} \sin^5(x) \cos^2(x) dx$$

48. Find the points on the parametric curve  $x(t) = t(t^2 - 3)$ ,  $y(t) = 3(t^2 - 3)$  where the tangent is horizontal and the points where it is vertical.

49. Find the values of  $x$  for which the series converges. Find the sum of the series for those values of  $x$

$$\sum_{n=1}^{\infty} \left( \frac{\ln x}{2} \right)^{2n}$$

[Hint: Reshape it into a geometric series.]

50. Use the integral test to show convergence and divergence of the following series:

(a) The convergence of

$$\sum_{n=1}^{\infty} \frac{1}{1+n^2}$$

(b) The divergence of

$$\sum_{n=1}^{\infty} \frac{\ln(n)}{n}$$

51. Find the interval and radius of convergence of the power series.

$$\sum_{i=1}^{\infty} \frac{(-1)^n 3^n}{n} (x-1)^n$$

52. (a) Evaluate the following indefinite integral as a power series:

$$\int x^2 e^{-x^2} dx$$

(b) Why does the series have an infinite radius of convergence?

53. Find the area bounded by  $\begin{cases} y = x^3 - x \\ y = 3x \end{cases}$

54. If  $f'(x) = e^{2x}$  and  $f(0) = 5$ , find  $f(10)$ .

55. If  $0 \leq f'(x)$  for  $-5 \leq x \leq 5$  and if the area bounded by  $y = f'(x)$ ,  $x = -5$ ,  $x = 5$  and  $y = 0$  is 29, find:

(a)  $f(5)$  if  $f(-5) = 10$ .

(b)  $\int_{-5}^5 |f'(x)| dx$

56. Find the radius of convergence of

(a)  $\sum_{n=0}^{\infty} \frac{4}{3^n(n!)} x^{2n}$

(b)  $\sum_{n=0}^{\infty} \frac{4^n x}{3^n}$

57. Determine if the following series are absolutely convergent:

(a)  $\sum_{n=0}^{\infty} \frac{4n+1}{(2n+1)!}$

(b)  $\sum_{n=0}^{\infty} \frac{4^n}{3^n}$

(c)  $\sum_{n=1}^{\infty} \frac{1}{4^n - 2^n}$

58. Determine if

$$\int_5^{\infty} \frac{1}{x} dx$$

converges or diverges.

59. Find the radius of convergence of

$$\sum_{n=1}^{\infty} \frac{n}{3^n} x^n.$$

60. Let  $f(x) = \frac{1}{1+x^2}$ .

(a) Find a power series,  $\sum_{n=0}^{\infty} a_n x^n$  whose sum is equal to  $f(x)$  for  $|x| < 1$ . [Hint: Use infinite geometric series.]

(b) Find a power series whose sum is  $\tan^{-1} x$  for  $|x| < 1$ . [Hint:  $\int \frac{1}{1+x^2} dx = \tan^{-1} x$ .]

61.  $F(x) = \int_3^{3x} e^{t^2} dt$ ,

(a) Find  $F'(x)$ .

(b)  $F'(0)$

(c)  $F(1)$

62. Let  $C$  be the curve described by  $\begin{cases} x = e^t + t^2 \\ y = \sin t + 2t - 3 \end{cases}$

(a) Find  $\frac{dy}{dx}$ .

- (b) Find  $\frac{ds}{dt}$  ( $s$  stands for arc length).
  - (c) Find the length of the arc from  $t = 0$  to  $t = 1$ .
  - (d) Find an equation of the line tangent to the curve  $C$  at  $(1, -3)(t = 0)$
63. Show that  $\sum \frac{4n^3}{2n^3}$  is divergent.
64. Find  $\int \frac{e^x dx}{e^{2x} + 1}$ .