Mathematics 131: Final, May 22, 2001

1) Consider the function

$$h(t) = \begin{cases} -t^2 + 2t + 1, & t \leq -1 \\ \frac{kt}{t^2+1}, & -1 < t < 1 \\ t^2 - 2t + 3, & t \geq 1. \end{cases}$$

A. Find the value of k that makes $h(t)$ a continuous function. Explain!

B. Is $h(t)$ (with this value of k) differentiable at $t = 1$? Explain!

2) Let

$$f(x) = \frac{7x^2 + 4x}{2x^2 - x + 3}.$$

A. Evaluate $\lim_{x\to\infty} f(x)$.

B. What is the horizontal asymptote to the graph of $f(x)$?

3. A waterskier skis over a straight ramp. Her speed is 13 m/s in the direction of the slope of the ramp (i.e., not horizontally). If the ramp is 12m long and 5m high, how fast is she rising when she leaves the ramp?

4. Use L'Hopital's Rule to find the value of the limit

$$\lim_{x\to 0}(1 + xe^x)^{1/\sin(x)}$$

5. For the function $f(x) = x^4 - 8x^2$, find

A. The intervals where the function is increasing and decreasing.

B. All local maxima and minima

C. The intervals on which the function is concave up and down, and its inflection points.

D. Sketch the graph, clearly indicating these points and intervals.

6. Consider the function $f(x) = (x^2 - 4x + 3)^{2/3}$ defined on the closed interval $0 \leq x \leq 5$.

A. Find all critical points of $f(x)$ on the interval $0 \leq x \leq 5$.

B. Find the global (absolute) minimum and global (absolute) maximum of $f(x)$ on this interval.

7. A square piece of tin 18 cm on a side is to be made into a rectangular box without a top by cutting a square from each corner and then folding up the flaps to form the sides. What size corners should be cut in order that the volume of the box be as large as possible?

Mathematics 131: Final, December 19, 2001

1) Consider the function

$$f(x) = \begin{cases} x^2 + x + k, & x \leq 0 \\ ke^x, & 0 < x < 1 \\ kex, & x \geq 1. \end{cases}$$

A. Show that $f(x)$ is continuous for all values of k.

B. Determine the value of k so that the function $f(x)$ is differentiable for all x.

2) Find the points on the ellipse $2x^2 + y^2 = 1$ where the tangent line has slope 1.

3) If a (perfectly round) snowball melts so that its surface area decreases at a rate of 2 cm^2/min, find the rate at which the diameter decreases when the diameter is 5 cm. (Hint: surface area A of a ball $A = 4\pi r^2$ where r is the radius of the ball.)

4) Find the following limits

A. $\lim_{x\to 0} \frac{\sqrt{5+x}-\sqrt{5}}{x}$

B. $\lim_{x\to 0}[1 + \sin(2x)]^{1/x}$

5) Consider the function $f(x) = x^3 - 6x^2 + 9x - 3$ defined on the interval $[0, 4]$.

A. Find the critical points of $f(x)$ on $[0, 4]$ and decide if the critical points are local maxima or minima.

B. Find the absolute maximum and minimum values of $f(x)$ on $[0, 4]$.

C. Determine the intervals in the domain of $f(x)$ on which $f(x)$ is concave up.

6) Let $g(x) = (x^2 + 2x - 2)e^{-x}$ be defined on the interval $[-3, 4]$

A. The graph of $g(x)$ has two inflection points in $[-3, 4]$. Find both of them.

B. Find the two intervals in the domain of $g(x)$ on which $g(x)$ is decreasing.

7) A car starts 10 miles south of an intersection and travels north at 20 miles per hour. At the same time, a second car starts 20 miles east of the intersection and travels west at 10 miles per hour. Neither car stops at the intersection. At what time are the two cars as close together as possible, and how far are they then?

8) Find the dimensions of the rectangle with largest area, so that its base is on the x-axis and two of the other vertices are on $y = 8 - x^2$ and have positive y-coordinates.