DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS
EXAM 1: MATH 131 Spring 2003
12 March 2003

Your Name: _____

Your Instructor's Name: _____

This exam paper consists of 10 questions, all of equal weight. It has 9 pages.
On this exam, you may use a calculator, but no books or notes.
It is not sufficient to just write the answers. You must *explain* how you arrive at your answers.

1. (10) _____
2. (10) _____
3. (10) _____
4. (10) _____
5. (10) _____
6. (10) _____
7. (10) _____
8. (10) _____
9. (10) _____
10. (10) _____

TOTAL (100)

(1) Compute the following limits.

(a) Explain each step with the limit laws. No credit will be gven for alternative solutions.

$$\lim_{x\to 1} e^x \frac{1}{1+x}$$

(b) Find the following limit $\lim_{x\to 2} \frac{(x^2-4)}{x(x-2)}$.

(2) Consider the function $f(x) = \begin{cases} x^2 - 1 & x \leq 1 \\ x - 1 & x > 1 \end{cases}$.
Is this function continuous at $x = 1$? Explain.

(3) Find $\lim_{x\to\infty} \dfrac{\sqrt{4x^2+1}}{2x+5}$.

(4) Find all horizontal and vertical asymptotes of

$$f(x) = \frac{(x^2+1)(x-1)}{(x^2-3x-2)}.$$

Show all the analytical steps involved.

(5) State the definition of the derivative of a function $f(x)$ at $x = a$.

(6) Using the definition of the derivative, find $f'(2)$ where $f(x) = x^2 - 3$.

(7) At $t = 0$ seconds, a baseball is thrown vertically upward from a window that is 160 feet above the ground. The height in feet of the baseball above the ground is given by the formula

$$h(t) = -16t^2 + Bt + A,$$

where A and B are some constants.

(a) Determine the value of the constant A.

(b) The ball reaches its highest point at the time $t = 1$. Use this information to determine the value of the constant B. (Hint: What is the velocity of the ball at the highest point?)

(c) At what time will the ball hit the ground?

(8) For what values of x is the tangent line to the curve $y = x^3 - x^2 - x + 1$ horizontal?

(9) State the quotient rule for the derivative of the function $y = f(x)/g(x)$.

(10) Find y', where $y = \dfrac{e^x + x}{x - 2}$.