

Fall '01 - Exam 1

(1) (15 pts)

- (a) The following is a table of values for the function $f(x) = 2x/(x^2 + 1)$. Compute the slopes of the secant lines through each of these points and the point $(0, 0)$. Use your table to estimate the slope of the tangent line to the graph $y = f(x)$ at $(0, 0)$.

x	0.1	0.01	0.001	0.0001
$f(x)$	0.198020	0.019998	0.002000	0.000200

- (b) Using your slope estimate from problem (a) above, write down the equation of the tangent line to $y = f(x)$ at the point $(0, 0)$.

(2) (15 pts) Given the function $f(x) = x/(x^2 - x - 2)$

- (a) Find the equations of all horizontal and vertical asymptotes of the graph.
(b) Draw the graph of $y = f(x)$, showing all the asymptotes.

(3) (16 pts) Let $f(x) = \sqrt{x - 1}$.

- (a) Calculate the derivative $f'(x)$ directly from the definition.
(b) Find the domain of $f(x)$ and of $f'(x)$.

(4) (24 pts) Consider the function

$$g(x) = \begin{cases} 3x, & -2 \leq x < -1 \\ 2 + x^2, & -1 \leq x < 0 \\ e^x + 1, & 0 \leq x \leq 1 \end{cases}$$

- (a) Find all points x at which $g(x)$ is discontinuous. Explain.
(b) Find all points x at which $g(x)$ is not differentiable. Explain.
(c) Draw the graph, clearly labelling these points.
(d) Evaluate the limit

$$\lim_{x \rightarrow 25} \frac{25 - x}{5 - \sqrt{x}}$$

showing all your steps clearly.

(5) (10 pts) Find the points on the graph of $y = x(x - 1)(x - 2)$ where the tangent line is horizontal.

(6) (10 pts) Calculate the derivative of the function

$$f(x) = \frac{x - k^2}{\sqrt{x - k}}$$

where k is a constant.