

DEPARTMENT OF MATHEMATICS AND STATISTICS
UNIVERSITY OF MASSACHUSETTS
MATH 131 Fall 2002
EXAM 2

Your Name: _____

Your Instructor's Name: _____

This exam paper consists of 7 questions. It has 9 pages.

On this exam, you may use a calculator, but no books or notes.

It is not sufficient to just write the answers. You must *explain* how you arrive at your answers.

If you draw a graph, you must include the value of the range variables and show the tick marks on the axes, if any.

If your drawing includes the graphs of more than one function, you must label them so that we can tell which is which.

1. (15) _____

2. (15) _____

3. (15) _____

4. (15) _____

5. (10) _____

6. (15) _____

7. (15) _____

TOTAL (100)

1. a) (10) Find the derivative $\frac{dy}{dx}$ of the function y , implicitly defined by the equation

$$(x + 2y)^2 = 3x^2y + 6x.$$

- b) (5) Find an equation of the tangent line at the point $(1, 1)$.

2. (15) A helicopter is hovering in place 300 meters above a highway. A radar on the helicopter measures the rate of change of the distance between the helicopter and approaching cars. What is the measured rate, for a car driving at a speed of 100 kilometers per hour, when it is 500 meters from the helicopter?

Note: 1 kilometer = 1000 meters.

3. (15) a) Find the linearization of the function $f(x) = \sqrt{x}$ at the point $a = 36$.

b) Use the linearization to approximate $\sqrt{35}$.

c) Is your approximation greater than or less than the actual value? **Justify your answer.**

4. (15) Let $f(x) = 0.25x^4 + x^3 - 5x^2$.
- a) Find the intervals on which $f(x)$ is increasing and those on which it is decreasing.

- b) Find all the local maxima and local minima of f .

c) Find the values of the global maximum and global minimum of f over the interval $-6 \leq x \leq 4$.

d) Graph the function over the interval $-6 \leq x \leq 4$, indicating all the local and global extrema.

5. (10) A particle is said to undergo a simple harmonic motion, if its position $s(t)$ at time t is given by the formula

$$s(t) = A \cos(\omega t + \delta)$$

for some constant real numbers A , ω , and δ , with $A > 0$ and $\omega > 0$.

- a) Express the acceleration $a(t)$ in terms of the time variable t and the constants A , ω , and δ .

- b) Assume now, that $s(t) = 3 \cos(2t)$, (so $A = 3$, $\omega = 2$ and $\delta = 0$). Find the time intervals, where both the position $s(t)$ and velocity $v(t)$ are positive.

- c) Is the particle in part b) speeding up or slowing down, when both its position and velocity are positive? **justify your answer.**

6. (15) Consider the function $f(x) = \ln(x^2 + 1)$.
- a) Determine the intervals on which the function is concave upward and those on which it is concave downward.

b) Determine all points of inflection, if any. **Justify your answer!**

7. (15) Use logarithmic differentiation to find the derivative $\frac{dy}{dx}$ for

$$y = \frac{\sqrt{x}(x^2 + 1)^7}{(x^x)}$$