

Mathematics 131: 2nd midterm, April 26, 2001

1) A squash ball is hit upwards so that its height in meters is given by  $h(t) = 5t - 10t^2$ , where  $t$  is the elapsed time in seconds.

- A. (10 pts) Find the velocity after 0.1 sec, 0.2 sec and 0.3 sec, respectively.
- B. (10 pts) When will the velocity be zero, and how high will the ball be then?
- C. (10 pts) At what time will the ball first hit the ground? With what velocity will it do so?

2) Let  $f(x) = e^{2x} \cos(x)$ .

- A. (10 pts) Find the derivative  $f'(x)$ .
- B. (10 pts) Find the equation of the tangent line to the graph of  $y = f(x)$  at the point  $(0, 1)$ .

3) The function  $f$  is given by  $f(t) = te^t$ .

- A. (10 pts) Find its first and second derivatives.
- B. (10 pts) Give a formula for the  $n$ -th derivative  $f^{(n)}(t)$ .

4) (10 pts) Use logarithmic differentiation to find the derivative of  $g(x) = x^{\ln(x)}$ , giving your answer in terms of  $x$ .

5) (10 pts) Find  $\frac{dy}{dx}$  when  $y$  is given implicitly by  $x^2y + xy^3 = 1$ .

6)

- A. (10 pts) Find the linearization of the function  $\sqrt{x}$  at  $a = 16$ .
- B. (10 pts) Use differentials to approximate  $\sqrt{16.5}$ .

7) (20 pts) A (perfectly cubic) chunk of ice is melting so that its volume decreases at a rate of  $15 \text{ cm}^3/\text{min}$ . At what rate is the side of the cube changing when that side is 10cm long?

Mathematics 131: 2nd midterm, Fall, 2001

1) A car position as a function of time is given by the formula  $s = f(t) = 2t^3 - 9t^2 + 12t$  where  $s$  is measured in feet and  $t$  is measured in seconds.

- A. (8 pts) What is the velocity of the car at time  $t$ ? When is the car at rest?
- B. (8 pts) When is the car moving in the positive direction? In the negative direction?
- C. (8 pts) Find the total distance the car travels in the first 3 seconds.

2) Suppose a function is defined implicitly by  $(x^2 + y^2)^2 = x^4 + 3$ .

- A. (8 pts) Find  $\frac{dy}{dx}$
- B. (6 pts) Find an equation of the tangent line to the curve at the point  $(1, 1)$ .

3) (12 pts) The function  $f$  is given by  $f(x) = e^{6x}$

- A. Find its first and second derivatives
- B. Give a formula for the  $n$ th derivative  $f^{(n)}(x)$ .

4) (12 pts) Use logarithmic differentiation to find the derivative of

$$f(x) = (x + 1)^2(x + 2)^3(x + 3)^4.$$

Gives your answers in terms of  $x$ .

5) (12 pts) Let  $f$  and  $g$  be differentiable functions, and let  $h(x) = \ln\left(\frac{f(x)}{\sqrt{g(x)}}\right)$ . If  $f(0) = 3$ ,  $g(0) = 1$ ,  $f'(0) = 1$  and  $g'(0) = -1$ , find  $h'(0)$ .

6) (14 pts) A plane flying horizontally at an altitude of 1 mile and a speed of 500 miles per hour passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when this distance is 2 miles.

7) (12 pts)

- A. Approximate  $\sqrt{6}$  using linear approximation and the fact that  $\sqrt{6.25} = 2.5$ .
- B. Is your approximation greater than or less than the actual value? Why?