Let $\gamma(t) = (a(t), b(t))$, $t \in I$ (an open interval), be a smooth embedded curve in the $xz$-plane, and without loss of generality assume $a(t) > 0$ and $|\gamma'(t)| = 1$ (i.e. unit speed). Let $M \subset \mathbb{R}^3$ be the surface of revolution obtained by revolving the image of $\gamma$ about the $z$-axis. There is a natural parametrization of $M$ given by $(\theta, t) \mapsto (a(t) \cos \theta, a(t) \sin \theta, b(t))$, which gives rise to a local coordinate chart of $M$, called the $(\theta, t)$ coordinates. We endow $M$ with the natural metric $g$ induced from $\mathbb{R}^3$.

1. Compute the Christoffel symbols of $(M, g)$ in the $(\theta, t)$ coordinates.
2. Show that each meridian $\theta = \theta_0$ is a geodesic in $(M, g)$ by verifying the geodesic equations.
3. Give an alternative proof of (2) by geometric considerations (hint: using isometries of $(M, g)$).
4. Determine necessary and sufficient conditions for a latitude circle $t = t_0$ to be a geodesic in $(M, g)$. 