1. Let $M$ be a compact, connected 1-dimensional smooth manifold (without boundary). Show that $M$ is diffeomorphic to the 1-sphere $S^1$. (Hint: consider a suitable integral curve on $M$ to obtain a diffeomorphism between $M$ and $S^1$.)

2. Let $X$ be a smooth vector field on the unit ball $B^n(1) \subset \mathbb{R}^n$ which vanishes at the origin, i.e., $X(0) = 0$. Show that for any given $R > 0$, there is a $0 < r < 1$ such that for any $p \in B^n(r)$ (ball of radius $r$), there is an integral curve $\gamma_p : [0, R] \rightarrow B^n(1)$ of $X$ with initial value $\gamma_p(0) = p$. Show that for any $t \in [0, R]$, $\theta_t : B^n(r) \rightarrow B^n(1)$ defined by $\theta_t(p) = \gamma_p(t)$ is a diffeomorphism onto its image, which has the properties (i) $\theta_0 = Id$ and (ii) $\theta_t(0) = 0$ for all $t \in [0, R]$. 