1. Show that the image of $F : \mathbb{S}^2 \rightarrow \mathbb{R}^4$, $(x, y, z) \mapsto (x^2 - y^2, xy, xz, yz)$ is an embedded submanifold of dimension 2, which is diffeomorphic to $\mathbb{R}P^2$.

2. Show that in the definition of a Lie group $G$, the assumption $i : G \rightarrow G$, where $i : g \mapsto g^{-1}$, is smooth can be derived from the assumption that $m : G \times G \rightarrow G$ is smooth, where $m : (g, h) \mapsto gh$.

3. Let $\mathbb{HP}^n$ be the set of quaternion lines in the quaternion space $\mathbb{H}^{n+1}$. Use Quotient Manifold Theorem to show that $\mathbb{HP}^n$ is naturally a compact smooth manifold. What’s its dimension?

4. Let $M$ be the set of oriented 2-planes in $\mathbb{R}^4$. Prove (1) $G = SO(4)$ acts transitively on $M$; (2) let $p \in M$ be the oriented 2-plane $\mathbb{R}^2 \times \{0\}$, determine $G_p$ as a subgroup of $G$ and show it is closed. With (1) and (2) one concludes that $M$ has a unique smooth manifold structure with respect to which $M$ is a homogenous space.