1. Let $G$ be a Lie group. Prove that (1) the pushforward of the inversion map $i : G \to G$ at $e \in G$, $i_* : T_eG \to T_eG$, is equal to $-Id$, (2) the Lie algebra of $G$ is Abelian if $G$ is Abelian.

2. Classify $k$-dimensional Lie algebras up to isomorphisms for $k = 1, 2$, and find examples of Lie groups whose Lie algebras realize the $k$-dimensional Lie algebras with $k = 1, 2$.

3. (optional) Let $G$ be the Lie group of non-zero quaternions. Let $X_i, i = 0, 1, 2, 3$ be the left invariant vector fields on $G$ such that at $1 \in G$, $X_0 = 1$, $X_1 = i$, $X_2 = j$, $X_3 = k$. Compute the Lie bracket in terms of the basis $\{X_i\}$. Observing $S^3$ is a Lie subgroup of $G$ and $i, j, k$ is a basis of $T_1S^3$, use the above computation to show that the Lie algebra of $S^3$ is isomorphic to $R^3$ with the cross product as the Lie bracket.