HOMEWORK 1

1. Let $M$ be a topological manifold. Then
   (1) Every smooth atlas on $M$ is contained in a unique maximal smooth atlas.
   (2) Two smooth atlases determine the same smooth structure if and only if their union is a smooth atlas.
   Part (1) is proved in J. Lee (Lemma 1.10). Prove part (2).

2. Let $M$ be the subset of $M(3 \times 3, \mathbb{R})$ consisting of upper triangle $3 \times 3$ matrices with determinant 1, which is given with the subspace topology. Show that $M$ is naturally a smooth manifold. Furthermore, what is the dimension of $M$? How many connected components does $M$ have?

3. Show that the 1-dimensional complex projective space $\mathbb{C}P^1$, i.e., the set of complex lines in $\mathbb{C}^2$, is naturally a complex manifold. Show that as a smooth 2-manifold, it is the same as the 2-sphere $S^2$ with the standard smooth structure.

4. Show that the set of 2-planes in $\mathbb{R}^4$, denoted by $G_2(\mathbb{R}^4)$, is naturally a smooth 4-manifold.