In this exam there are 6 pages, including this one, and six problems. Make sure you have them all before you begin!

1. (10) ______________
2. (15) ______________
3. (20) ______________
4. (20) ______________
5. (20) ______________
6. (15) ______________
Total (100) ______________

Instructions:

- One page cheat sheet is allowed and a calculator is allowed.
- You must explain how you arrived at your answers, and show your algebraic calculations.
- Simplify your expressions! But please leave fractions and square roots in your answers and do not give decimal expansions.
- All of these expressions are acceptable ways to notate vectors: $xi + yj +zk$, $(x, y, z)$, $[x, y, z]$, $(x, y, z)$. 
1. Evaluate the double integral

\[ \iint_D 1 - 2(x^2 + y^2) \, dA \]

where \( D \) is a half disc \( x^2 + y^2 \leq 1 \) for \( x \geq 0 \).
2. Sketch the region of integration and evaluate the integral by changing the order of integration.

\[ I = \int_0^1 \int_x^1 \sin(y^2) \, dy \, dx \]
3. (a) Use Lagrange multipliers to find the extreme values of \( f(x, y) = x^2 + 2y^2 + 5 \) subject to the condition that \((x, y)\) lies on the ellipse \( x^2 + 4y^2 = 4 \).

(b) Using part (a), find the absolute maximum and absolute minimum values of \( f(x, y) \) inside the ellipse or, in other words, in the region where \( x^2 + 4y^2 \leq 4 \).
4. Find an equation of the plane that passes through the point \((2, 4, 1)\) and contains the line \(x = 5 - 3t, \ y = 2 + t, \ z = 4 - 6t.\)
5. Given $\mathbf{F}(x, y) = e^x \sin(y) \mathbf{i} + e^x \cos(y) \mathbf{j}$.

(a) Determine whether $\mathbf{F}$ is a conservative vector field.

(b) Find a potential function $f$ such that $\nabla f = \mathbf{F}$

(c) Evaluate the line integral

$$\int_{C} \mathbf{F}(x, y) \cdot d\mathbf{r}$$

where $C$ is a line segment from $(0, 0)$ to $(1, \frac{\pi}{2})$. 
6. Evaluate the given integral

\[ \oint_C \left( e^{3x} + x^2 y \right) dx + \left( e^{3y} - xy \right) dy \]

where C is the unit circle \( x^2 + y^2 = 1 \) oriented counterclockwise.