Here are some problems about improper integrals, approximate integration, and limits of sequences. These topics are not covered in the posted old Exam 2 copies (which were limited to methods of symbolic integration).

1. Determine whether the improper integral converges and, if so, find its value:

   (a) \( \int_{-\infty}^{1} xe^{-x^2} \, dx \)

   (b) \( \int_{1}^{\infty} \frac{1}{\sqrt{x(1 + x)}} \, dx \)

   (c) \( \int_{1}^{2} \frac{1}{(x - 1)^2} \, dx \)

   (d) \( \int_{0}^{2} \frac{x}{\sqrt{4 - x^2}} \, dx \)

2. For each of the following two methods, set up a sum that, if you did the arithmetic, would approximate \( \int_{0}^{1} \frac{1}{x^2 + 1} \, dx \):

   (a) The trapezoidal rule with \( n = 6 \) subintervals.

   (b) Simpson’s rule with \( n = 6 \) subintervals.

3. Determine whether the sequence \( \{a_n\}_{n=1}^{\infty} \) converges and, if so, find its limit:

   (a) \( a_n = \frac{3n^2 + 4n}{5 + 6n^2} \)

   (b) \( a_n = \frac{\sin n}{n^2} \)

   (c) \( a_n = \frac{1 + (-1)^n}{n + 2} \)

   (d) \( a_n = \frac{4^{n+1}}{7^{2n}} \)