1. (18 points) Compute the integral \( \int_C \overline{z} \, dz \), where \( C \) is the triangle with vertices at the points 0, 1, and \( i \), (traversed counterclockwise). Caution: The integrand is the complex conjugate \( \overline{z} \) of \( z \).

2. (18 points) Let \( C \) be the ellipse cut out by the equation \( \left( \frac{x}{3} \right)^2 + \left( \frac{y}{5} \right)^2 = 1 \), oriented counterclockwise. Compute \( \int_C \frac{z^3 \, dz}{(z - i)(z^2 + 1)} \).

3. (16 points) Suppose that \( f(z) \) is entire and \( |f(z)| \geq 1/2 \), for all \( z \) in the complex plane. Prove that \( f \) is a constant function. Hint: The strategy is similar to the proof of the Fundamental Theorem of Algebra, but the actual proof is much simpler.

4. (16 points) Let \( C \) be the unit circle parametrized by \( z(\theta) = e^{i\theta}, 0 \leq \theta \leq 2\pi \).

(a) Show that for all integers \( n \), \( \int_C \frac{e^{(zn)/z} \, dz}{z} = 2\pi i \).

(b) Derive the integration formula \( \int_0^{2\pi} e^{\cos(n\theta) \cos(n\theta)} d\theta = 2\pi \), for every integer \( n \).

5. (16 points) Let the domain \( D \) be the complex plane minus the non-negative part of the \( x \)-axis. Let \( \log(z) \) be the branch of the logarithm function with argument in the interval \( (0, 2\pi) \), so that \( \log(z) \) is analytic in \( D \). Set \( f(z) := e^{(1/2)\log(z)} \). Note that \( f(z) \) is a branch of the multi-valued function \( \sqrt{z} \).

(a) Find a single valued anti-derivative \( F(z) \) of \( f(z) \) in \( D \). Express your answer in terms of the above branch of \( \log(z) \) and avoid using multi-valued rational powers of \( z \). Check that your answer is indeed an anti-derivative, by explicitly differentiating it.

(b) Let \( C \) be the contour \( z(\theta) = e^{i\theta}, \pi/2 \leq \theta \leq 3\pi/2 \). Prove the equality
\[
\int_C f(z) \, dz = \frac{2\sqrt{2}}{3}.
\]

6. (16 points) Let \( C \) be a circle of radius \( 7/2 \) centered at the origin oriented counterclockwise. Set \( g(n) := \int_C \frac{z^5 + 3z + 7}{(z - n)^3} \, dz \). Compute \( g(n) \) for all integers \( n \). Justify your answer!!!