In this exam there are seven problems. Make sure you have them all. You can use a calculator, but only to perform arithmetic calculations. You must SHOW ALL WORK other than arithmetic calculations. Textbooks, class notes, formula sheets, etc. are NOT allowed. BOX your final answers.

1. (15) ____________
2. (10) ____________
3. (15) ____________
4. (15) ____________
5. (15) ____________
6. (15) ____________
7. (15) ____________
Total (100) ____________
1. Consider the lines $L_1$ and $L_2$ given by their symmetric equations

$$
L_1 : \frac{x - 3}{2} = \frac{y + 1}{-4} = \frac{z - 1}{3}
$$

and

$$
L_2 : \frac{x - 3}{1} = \frac{y - 5}{1} = \frac{z - 2}{2}.
$$

(a) Show that $L_1$ and $L_2$ intersect and find the point of intersection.

(b) Find an equation for the plane containing $L_1$ and $L_2$. 
2. A particle moves with position function

\[ r(t) = (t^2 - 8t)i + 2t^2j. \]

When is the speed a minimum?
3. The oil spill forms in the shape of a triangle with sides of length $x$, $y$, and $z$. According to Heron’s formula, its area is given by

$$A = \frac{1}{4} \sqrt{(x^2 + y^2 + z^2)^2 - 2(x^4 + y^4 + z^4)}.$$ 

Find the rate at which the area is changing if $x = 3$ miles and increases at a rate of 1 mph, $y = 4$ miles and decreases at a rate of 1 mph, $z = 5$ miles and does not change.
4. Find the absolute maximum and minimum values of the function $f(x, y) = x^2 + 2x + 2y^2$
on the disc $D = \{x^2 + y^2 \leq 4\}$ and the points at which these extreme values occur.
5. Find the volume of the solid bounded by the cylinder $x^2 + y^2 = 1$, the plane $z = 0$, and the elliptic paraboloid $z = x^2 + y^2 + 5$. 
6. Consider the vector field

\[ \mathbf{F}(x, y) = e^y \mathbf{i} + e^{2x} \mathbf{j}. \]

(a) Is this vector field conservative? If the answer is yes, find a function \( f(x, y) \) such that \( \mathbf{F}(x, y) = \nabla f(x, y) \).

(b) Compute the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \), where \( C \) is the line segment from \((1, 2)\) to \((3, 7)\).
7. Use Green’s Theorem to compute the line integral

$$\int_C \sin(x^2) \, dx + (x + y)^2 \, dy,$$

where $C$ is the positively oriented boundary of the region in the first quadrant enclosed by the lines $x = 0$, $y = 0$ and the parabola $y = 1 - x^2$. 
Scratch paper