1. Suppose $y \sim MVN(\mathbf{0}, \Sigma)$, $\Sigma = \begin{pmatrix} 1 & \rho \\ \rho & 1 \end{pmatrix}$, $-1 < \rho < 1$. Are $y_1^2 + 2y_1y_2 + y_2^2$ and $y_1^2 - 2y_1y_2 + y_2^2$ independent? Why or why not? You may cite a theorem without proof.

2. Let $y$ be a random vector with length $n$. Let $X$ be an $n$ by $p$ matrix with rank $p$. Suppose $y$ is normal with mean $X\beta$ and covariance $\sigma^2 I_n$.

   (a) What is the least squares estimator of $\beta$? (You do not need to derive this.)

   (b) What are matrix expressions for $\hat{y}$ and $e$ (residuals)? (You do not need to derive this.)

   (c) What is the distribution of the vector $t = (\hat{y}^T, e^T)^T$? (Please justify your answer.)

   (d) Are SSE and $\hat{\beta}$ independent? Why or why not?

3. Evaluate the integral $\int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} (x^T B x + x^T c) \exp \left\{ - (x - a)^T \Sigma^{-1} (x - a) \right\} dx_1, \ldots, dx_n$.

   For full credit, please justify each step.