Hw 3 (due Feb. 18, Thursday)

2.132
[Sol] Let define the following events,
\( F_i \) : the plane is found in region \( i \) when it is searched,
\( N_i \) : the plane is not found in region \( i \) when it is searched,
\( R_i \) : the plane is in region \( i \).
Then \( P(R_i) = 1/3 \) for all \( i \), \( P(F_i \mid R_i) = 1 - \alpha_i \) and \( P(N_i \mid R_i) = \alpha_i \).

(a) \( P(R_1 \mid N_1) = \frac{P(N_1 \cap R_1)}{P(N_1)} = \frac{P(N_1 \mid R_1)P(R_1)}{P(N_1)} = \frac{\alpha_i}{\alpha_i + \frac{1}{3}} = \frac{1}{\alpha_i + 2} \)

(b) \( P(R_2 \mid N_1) = \frac{P(N_1 \cap R_2)}{P(N_1)} = \frac{P(N_1 \mid R_2)P(R_2)}{P(N_1)} = \frac{\frac{1}{3}}{\alpha_i + \frac{1}{3} + \frac{1}{3}} = \frac{1}{\alpha_i + 2} \)

(c) \( P(R_3 \mid N_1) = \frac{P(N_1 \cap R_3)}{P(N_1)} = \frac{P(N_1 \mid R_3)P(R_3)}{P(N_1)} = \frac{\frac{1}{3}}{\alpha_i + \frac{1}{3} + \frac{1}{3}} = \frac{1}{\alpha_i + 2} \)

2.135
[Sol] Let define the following events ,
\( M \) : travelers fly on major airlines, \( P \) : travelers fly on privately owned planes, \( C \) : travelers fly on commercially owned planes, \( B \) : the person travels on business.
Then \( P(M) = .6, P(P) = .3, P(C) = .1, P(B \mid M) = .5, P(B \mid P) = .6 \) and \( P(B \mid C) = .9 \).

(a) \( P(B) = P(B \cap (M \cup P \cup C)) = P((B \cap M) \cup (B \cap P) \cup (B \cap C)) = P(B \cap M) + P(B \cap P) + P(B \cap C) = P(M) \times P(B \mid M) + P(P) \times P(B \mid P) + P(C) \times P(B \mid C) = .6 \times .5 + .3 \times .6 + .1 \times .9 = .57 \)

(b) \( P(P \cap B) = P(P) \times P(B \mid P) = .3 \times .6 = .18 \)

(c) \( P(P \mid B) = \frac{P(P \cap B)}{P(B)} = \frac{.18}{.57} = .3158 \)

(d) \( P(B \mid C) = .9 \)

3.2
[Sol] The simple events and corresponding \( Y \) are Since \( P(E_i) = 1/4 \) for each \( i \), the probability

<table>
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<tr>
<th>( E_i )</th>
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<tbody>
<tr>
<td>Y</td>
<td>2</td>
<td>-1</td>
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distribution for \( Y \) is

<table>
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<th>( y )</th>
<th>-1</th>
<th>1</th>
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<tbody>
<tr>
<td>( p(y) )</td>
<td>1/2</td>
<td>1/4</td>
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3.4
[Sol] Define the following events.
\( A \): valve 1 fails, \( B \): valve 2 fails, \( C \): valve 3 fails.
Then \( P(Y = 2) = P(A \cap B \cap C) = .8^3 = .512 \),
\( P(Y = 0) = P(A \cap (B \cup C)) = P(A)P(B \cup C) = P(A)(P(B) + P(C) - P(B \cap C)) = 0.2 \cdot (0.2 + 0.2 - 0.2 \cdot 0.2) = 0.072 \),
\( P(Y = 1) = 1 - P(Y = 2) - P(Y = 0) = 0.416 \).

3.9
[Sol] The random variable \( Y \) takes on the values 0, 1, 2, and 3. We can assume that the three entries are independent.
(a) Let \( E \) denote an error on a single entry; let \( N \) denote that there is no error. There are 8 sample points; \( EEE, EEN, ENE, NEE,ENN,NEN,NNE,NNN \).
Thus, \( P(Y = 3) = P(EEE) = 0.05^3 = 0.000125 \),
\( P(Y = 2) = P(EEN) + P(ENE) + P(NEE) = 3 \cdot 0.05^2 \cdot 0.95 = 0.007125 \),
\( P(Y = 1) = P(ENN) + P(NEN) + P(NNE) = 3 \cdot 0.05^2 \cdot 0.95^2 = 0.135375 \),
\( P(Y = 0) = P(NNN) = 0.95^3 = 0.857375 \).
(c) \( P(Y > 1) = P(Y = 2) + P(Y = 3) = 0.00725 \).

3.12
[Sol] \( E(Y) = \sum_y yp(y) = 1 \cdot 0.4 + 2 \cdot 0.3 + 3 \cdot 0.2 + 4 \cdot 0.1 = 2.0 \),
\( E(1/Y) = \sum_y (1/y)p(y) = 1 \cdot 0.4 + (1/2) \cdot 0.3 + (1/3) \cdot 0.2 + (1/4) \cdot 0.1 = 0.6417 \),
\( E(Y^2 - 1) = E(Y^2) - 1 = 1^20.4 + 4^20.3 + 9^20.2 + 16^20.1 - 1 = 4 \),
\( V(Y) = E(Y^2) - E(Y)^2 = 5 - 2^2 = 1 \).

3.24
[Sol] Consider the probability distribution for \( Y \):
\[
\begin{array}{ccc}
 y & 0 & 1 & 2 \\
p(y) & 0.81 = (0.9^2) & 0.18 = (0.9 \cdot 0.1 \cdot 2) & 0.01 = (0.1^2) \\
\end{array}
\]
Then \( \mu = E(Y) = \sum_y yp(y) = 0 \cdot 0.81 + 1 \cdot 0.18 + 2 \cdot 0.01 = 0.2 \),
\( \sigma^2 = V(Y) = E(Y^2) - \mu^2 = 0 \cdot 0.81 + 1 \cdot 0.18 + 4 \cdot 0.01 - 0.2^2 = 0.18 \).

3.34
[Sol] The mean cost is \( E(10Y) = 10E(Y) \). Now \( E(Y) = 0 \cdot 0.1 + 1 \cdot 0.5 + 2 \cdot 0.4 = 1.3 \),
so that the mean cost is $13. The variance of the cost is \( V(10Y) = 100V(Y) \). Now
\( V(Y) = E(Y^2) - E(Y)^2 = 0 \cdot 0.1 + 1 \cdot 0.5 + 4 \cdot 0.4 - 1.3^2 = 0.41 \), so that the variance of the cost is \( 100 \cdot 0.41 = 41 \).