Extra-credit practice problems

---

1. Find the first and second derivatives, i.e., $f'(x)$ and $f''(x)$, respectively of the functions given:

   (a) $f(x) = x \sin(1/x)$

   (b) $f(x) = [\ln(1 + e^x)]^2$

   (c) $f(x) = \dfrac{\ln x}{x^2}$

   (d) $f(x) = x^{\sin x}$

2. Compute each of the partial derivatives $f_x = \dfrac{\partial f}{\partial x}$, $f_{xx} = \dfrac{\partial^2 f}{\partial x^2}$, $f_y = \dfrac{\partial f}{\partial y}$, $f_{yy} = \dfrac{\partial^2 f}{\partial y^2}$ and $f_{xy} = \dfrac{\partial^2 f}{\partial x \partial y}$ for the following functions $f = f(x, y)$:

   (a) $f(x, y) = y \ln y - e^{xy^2}$

   (b) $f(x, y) = \ln(xy) + \dfrac{x^2 + 1}{y - 5}$

   (c) $f(x, y) = (x^2 + y^3)^{10} + \ln x$

   (d) $f(x, y) = \sin(xy) \cos(x^2)$

3. Evaluate the indefinite integrals:

   (a) $\displaystyle\int x \cos x \, dx$

   (b) $\displaystyle\int \ln x \, dx$

   (c) $\displaystyle\int \dfrac{\ln x}{x} \, dx$

   (d) $\displaystyle\int \dfrac{1 + 4x}{\sqrt{1 + x + 2x^2}} \, dx$

   (e) $\displaystyle\int x \sin x \, dx$

   (f) $\displaystyle\int x^2 e^x \, dx$

   (g) $\displaystyle\int \dfrac{4}{t^2 + 5t - 14} \, dt$

   (h) $\displaystyle\int \dfrac{8 - 3t}{10t^2 + 13t - 3} \, dt$

4. Evaluate the definite integrals for $s > 0$, $a \in \mathbb{R}$:

(a) $\int_0^\infty e^{-st} dt$

(b) $\int_0^\infty t e^{-st} dt$

(c) $\int_0^\infty t^2 e^{-st} dt$

(d) $\int_0^\infty t^n e^{-st} dt$, $n \in \mathbb{N}$

(e) $\int_0^\infty \sin(at) e^{-st} dt$

(f) $\int_0^\infty \cos(at) e^{-st} dt$

(g) $\int \frac{1}{2t^2 - 3t + 2} dt$ Hint: Complete the square!

5. Write the system of equations in matrix notation $A\mathbf{x} = \mathbf{b}$ and find the values of x, y, and z such that these equations hold.

$$x - 2y + 3z = 7$$
$$-x + y - 2z = -5$$
$$2x - y - z = 4$$

6. For 2x2 matrices, $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, we define the **determinant**, $\Delta = ad - bc$ and the **trace**, $\tau = a + d$. Determine the determinant and the trace of the following matrices.

(a) $\begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}$

(b) $\begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix}$

(c) $\begin{pmatrix} 3 & \sqrt{2} \\ \sqrt{2} & -2 \end{pmatrix}$

(d) $\begin{pmatrix} -\frac{1}{2} & 1 \\ -1 & -\frac{1}{2} \end{pmatrix}$

(e) $\begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}$

(f) $\begin{pmatrix} 7 & -3 \\ 5 & 5 \end{pmatrix}$

(g) $\begin{pmatrix} 7 & 3 \\ 5 & 5 \end{pmatrix}$

(h) $\begin{pmatrix} 14 & -2 \\ 7 & -1 \end{pmatrix}$

(i) $\begin{pmatrix} 2 & 4 \\ 8 & 12 \end{pmatrix}$

(j) $\begin{pmatrix} 0 & 2 \\ 0 & 1 \end{pmatrix}$