Instructions: Please make sure to demonstrate every step in your calculations. Return your answers **including this homework sheet** back to the instructor **as a single, stapled package**. Also, please keep a copy of your solutions for your reference, especially in view of studying for exams.

Name: ______________________________

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1. Write the following differential equations as first order systems. State whether the system is linear or nonlinear.
   (a) $y''' + 4t^2 y'' - y' = 0$
   (b) $y'' + y' - y^2 = 0$
   (c) $y'''' - 2y = 5e^{-t}$

Solution:

(a) First rewrite the differential equation as $y''' = y' - 4t^2 y''$. Now define $x_1 = y, x_2 = y'$ and $x_3 = y''$. Then we have the system

$$\begin{aligned} x_1' &= (y)' = x_2 \\ x_2' &= (y')' = x_3 \\ x_3' &= (y'')' = y' - 4t^2 y'' = x_2 - 4t^2 x_3 \end{aligned}$$

This is a linear system with the coefficient matrix

$$\mathbf{P}(t) = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & -4t^2 \end{pmatrix}$$

and $\mathbf{g}(t) = \mathbf{0}$.

(b) Define $x_1 = y$ and $x_2 = y'$. Then we have

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= x_1^2 - x_2 \end{aligned}$$

This is a nonlinear system.

(c) Define $x_1 = y, x_2 = y', x_3 = y''$ and $x_4 = y'''$. Then we have the system

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= x_3 \\ x_3' &== x_4 \\ x_4' &= 2x_1 + 5e^{-t} \end{aligned}$$

This is a linear system with

$$\mathbf{P}(t) = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 0 & 0 & 0 \end{pmatrix}, \quad \mathbf{g}(t) = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 5e^{-t} \end{pmatrix}$$

2. Let $x = x_1(t)$, $y = y_1(t)$ and $x = x_2(t)$, $y = y_2(t)$ be any two solutions of the linear nonhomogeneous system

$$\begin{aligned} x' &= p_{11}(t)\,x + p_{12}(t)\,y + g_1(t), \\ y' &= p_{21}(t)\,x + p_{22}(t)\,y + g_2(t). \end{aligned}$$

Show that $x = x_1(t) - x_2(t)$, $y = y_1(t) - y_2(t)$ is a solution of the corresponding homogeneous system.

Solution: To show this, let us consider the homogeneous system of Eq. (7), i.e., with $g_1(t) = g_2(t) = 0\ \forall t$. Then, direct substitution of $x = x_1(t) - x_2(t)$, $y = y_1(t) - y_2(t)$ into the homogeneous system yields to

$$\begin{aligned} (x_1 - x_2)' &= p_{11}(t)\,(x_1 - x_2) + p_{12}(t)\,(y_1 - y_2), \\ (y_1 - y_2)' &= p_{21}(t)\,(x_1 - x_2) + p_{22}(t)\,(y_1 - y_2), \end{aligned}$$

and upon reshuffling terms we obtain

$$\begin{aligned} x_1' - (p_{11}(t)x_1 + p_{12}(t)y_1) &= x_2' - (p_{11}(t)x_2 + p_{12}(t)y_2)\,, \\ y_1' - (p_{21}(t)x_1 + p_{22}(t)y_1) &= y_2' - (p_{21}(t)x_2 + p_{22}(t)y_2)\,. \end{aligned}$$

Next, note that the left- and right-hand-sides equal to $g_1(t)$ and $g_2(t)$ for the first and second equations, respectively (recall that $x = x_1(t)$, $y = y_1(t)$ and $x = x_2(t)$, $y = y_2(t)$ are solutions to the nonhomogeneous system). This way, we conclude that $x = x_1(t) - x_2(t)$, $y = y_1(t) - y_2(t)$ is indeed a solution of the corresponding homogeneous system.

3. For the following matrices compute the eigenvalues and eigenvectors and indicate the type of the system (saddle point, node (or sink), source, center, spiral source, spiral sink).

(a)

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix}$$

(b)

$$\mathbf{A} = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$$

(c)

$$\mathbf{A} = \begin{pmatrix} 3 & -4 \\ 1 & 3 \end{pmatrix}$$

Solution: Recall that we determine the eigenvalues via the characteristic equation:

$$\det(A - r\mathbb{I}) = 0. \tag{1}$$

Then, upon finding the eigenvalues, we will determine the components of the eigenvectors, i.e., x_1 and x_2 of $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$

(a) The eigenvalues of the first matrix are $r_1 = 1$ and $r_2 = 3$. For r_1 the eigenvector can be found as follows:

$$\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \Rightarrow x_1 = x_2.$$

Thus, for $x_1 = 1$, the associated eigenvector is $\mathbf{x}^{(1)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$. On equally footing and for r_2 at hand, we have

$$\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 3 \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \Rightarrow -x_1 = x_2.$$

For $x_1 = 1$, we obtain $\mathbf{x}^{(2)} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

(b) The eigenvalues of the second matrix are $r_1 = 2i$ and $r_2 = -2i$. The associated eigenvector $\mathbf{x}^{(1)}$ for r_1 can be obtained via

$$\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 2i \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \Rightarrow -2x_2 = 2ix_1.$$

For $x_1 = 1$ we obtain $\mathbf{x}^{(1)} = \begin{pmatrix} 1 \\ -i \end{pmatrix}$. Finally, and as per the second eigenvalue λ_2, we have similarly

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = -i \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \Rightarrow x_1 = -ix_2,$$

and for $x_1 = 1$, the eigenvector is $\mathbf{x}^{(2)} = \begin{pmatrix} 1 \\ i \end{pmatrix}$.

(c) The eigenvalues of the third matrix are $r_1 = 3 + 2i$ and $r_2 = 3 - 2i$. The associated eigenvector $\mathbf{x}^{(1)}$ for r_1 can be obtained via

$$\begin{pmatrix} 3 & -4 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = (3 + 2i) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \Rightarrow 2ix_2 = x_1.$$

For $x_2 = 1$ we obtain $\mathbf{x}^{(1)} = \begin{pmatrix} (2i \\ 1 \end{pmatrix}$. Finally for the second eigenvalue λ_2, the eigenvector is $\mathbf{x}^{(2)} = \begin{pmatrix} -2i \\ i \end{pmatrix}$.

4. For the system of differential equations

$$\mathbf{x}' = \mathbf{A}\mathbf{x} \quad \mathbf{A} = \begin{pmatrix} 2 & -5 \\ 1 & -2 \end{pmatrix}$$

and the two vector-valued functions

$$\mathbf{x}_1(t) = \begin{pmatrix} 5\cos t \\ 2\cos t + \sin t \end{pmatrix}, \quad \mathbf{x}_2(t) = \begin{pmatrix} 5\sin t \\ 2\sin t - \cos t \end{pmatrix}$$

do the following:

(a) Show that the given functions are solutions of the given system of differential equations.

(b) Show that $\mathbf{x} = c_1\mathbf{x}_1 + c_2\mathbf{c}_2$ is also a solution of the given system for any value of c_1 and c_2.

(c) Find the solution of the given system that satisfies the initial condition $\mathbf{x}(0) = (1,2)^T$

Solution:

(a) Compute

$$\mathbf{x}_1' = \begin{pmatrix} -5\sin t \\ -2\sin t - \cos t \end{pmatrix}, \quad \mathbf{x}_2' = \begin{pmatrix} 5\cos t \\ 2\cos t + \sin t \end{pmatrix}$$

and

$$\mathbf{A}\mathbf{x}_1 = \begin{pmatrix} 10\cos t - 10\cos t - 5\sin t \\ 5\cos t - 4\cos t - 2\sin t \end{pmatrix}, \quad \mathbf{A}\mathbf{x}_2 = \begin{pmatrix} 10\sin t - 10\sin t + 5\cos t \\ 5\sin t - 4\sin t + 2\cos t \end{pmatrix}$$

Upon simplifying, we clearly have the equalities $\mathbf{A}\mathbf{x}_1 = x_1'$ and $\mathbf{A}\mathbf{x}_2 = \mathbf{x}_2'$.

(b) Using (a), we have

$$\begin{aligned} \mathbf{A}\mathbf{x} &= \mathbf{A}(c_1\mathbf{x}_1 + c_2\mathbf{x}_2) \\ &= c_1\mathbf{A}\mathbf{x}_1 + c_2\mathbf{A}\mathbf{x}_2 \\ &= c_1\mathbf{x}_1' + c_2\mathbf{x}_2' \\ &= (c_1\mathbf{x}_1 + c_2\mathbf{x}_2)' \\ &= \mathbf{x}' \end{aligned}$$

(c) We need to find c_1 and c_2 such that

$$c_1\mathbf{x}_1(0) + c_2\mathbf{x}_2(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

This gives us the two equations

$$\begin{aligned} 5c_1 &= 1, \\ 2c_1 - c_2 &= 2 \end{aligned}$$

which has solution $c_1 = 1/5$ and $c_2 = -8/5$. So

$$\mathbf{x}(t) = \frac{1}{5}\mathbf{x}_1 - \frac{8}{5}\mathbf{x}_2$$

5. Solve the given initial value problem. Describe the behavior of the solution as $t \to \infty$

$$\mathbf{x}' = \begin{pmatrix} 5 & -1 \\ 3 & 1 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$$

Solution: Firstly, finding the eigenvalues and eigenvectors of the coefficient matrix A as follows:

$$\det(A - r\mathbb{I}) = \begin{vmatrix} 5-r & -1 \\ 3 & 1-r \end{vmatrix} = 0 \Rightarrow r^2 - 6r + 8 = (r-2)(r-4) = 0$$

The two eigenvalues are $r_1 = 2$ and $r_2 = 4$.

For $r_1 = 2$, the corresponding eigenvector $\boldsymbol{\xi}^{(1)} = (\xi_1^{(1)}, \xi_2^{(1)})^T$ can be found by solving

$$\begin{pmatrix} 5-2 & -1 \\ 3 & 1-2 \end{pmatrix} \begin{pmatrix} \xi_1^{(1)} \\ \xi_2^{(1)} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Hence, $3\xi_1^{(1)} = \xi_2^{(1)}$. So the eigenvector $\boldsymbol{\xi}^{(1)}$ corresponding to the eigenvalue $r_1 = 2$ is

$$\boldsymbol{\xi}^{(1)} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

Similarly, for $r_2 = 4$, the corresponding eigenvector $\boldsymbol{\xi}^{(2)} = (\xi_1^{(2)}, \xi_2^{(2)})^T$ can be found by solving

$$\begin{pmatrix} 5-4 & -1 \\ 3 & 1-4 \end{pmatrix} \begin{pmatrix} \xi_1^{(2)} \\ \xi_2^{(2)} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Hence, $\xi_1^{(2)} = \xi_2^{(2)}$. So the eigenvector $\boldsymbol{\xi}^{(2)}$ corresponding to the eigenvalue $r_2 = 4$ is

$$\boldsymbol{\xi}^{(2)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Thus a fundamental set of solutions of the system is

$$\mathbf{x}^{(1)}(t) = \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{2t}, \quad \mathbf{x}^{(2)}(t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t}$$

The general solution is

$$\mathbf{x}(t) = c_1\mathbf{x}^{(1)}(t) + c_2\mathbf{x}^{(2)}(t) = c_1 \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{2t} + c_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t}$$

Plug in initial value $\mathbf{x}(0) = (2, -1)^T$ and solve for c_1 and c_2

$$c_1 = -\frac{3}{2}, \quad c_2 = \frac{7}{2}$$

Thus the particular solution for the initial value problem is

$$\mathbf{x}(t) = -\frac{3}{2} \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{2t} + \frac{7}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t}$$

As $t \to \infty$, the second term $\frac{7}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{4t}$ is dominant and the first term $-\frac{3}{2} \begin{pmatrix} 1 \\ 3 \end{pmatrix} e^{2t}$ is negligible. $\mathbf{x}(t)$ becomes asymptotic to the line $x_2 = x_1$ as $t \to \infty$

6. Solve the given initial value problem. Describe the behavior of the solution as $t \to \infty$

$$\mathbf{x}' = \begin{pmatrix} -2 & 1 \\ -5 & 4 \end{pmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

Solution: Firstly, finding the eigenvalues and eigenvectors of the coefficient matrix A as follows:

$$\det(A - r\mathbb{I}) = \begin{vmatrix} -2-r & 1 \\ -5 & 4-r \end{vmatrix} = 0 \Rightarrow r^2 - 2r - 3 = (r+1)(r-3) = 0$$

The two eigenvalues are $r_1 = -1$ and $r_2 = 3$.
For $r_1 = -1$, the corresponding eigenvector $\boldsymbol{\xi}^{(1)} = (\xi_1^{(1)}, \xi_2^{(1)})^T$ can be found by solving

$$\begin{pmatrix} -2-(-1) & 1 \\ -5 & 4-(-1) \end{pmatrix} \begin{pmatrix} \xi_1^{(1)} \\ \xi_2^{(1)} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Hence, $\xi_1^{(1)} = \xi_2^{(1)}$. So the eigenvector $\boldsymbol{\xi}^{(1)}$ corresponding to the eigenvalue $r_1 = -1$ is

$$\boldsymbol{\xi}^{(1)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Similarly, for $r_2 = 3$, the corresponding eigenvector $\boldsymbol{\xi}^{(2)} = (\xi_1^{(2)}, \xi_2^{(2)})^T$ can be found by solving

$$\begin{pmatrix} -2-3 & 1 \\ -5 & 4-3 \end{pmatrix} \begin{pmatrix} \xi_1^{(2)} \\ \xi_2^{(2)} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

Hence, $\xi_1^{(2)} = \xi_2^{(2)}$. So the eigenvector $\boldsymbol{\xi}^{(2)}$ corresponding to the eigenvalue $r_2 = 3$ is

$$\boldsymbol{\xi}^{(2)} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$$

Thus a fundamental set of solutions of the system is

$$\mathbf{x}^{(1)}(t) = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t}, \quad \mathbf{x}^{(2)}(t) = \begin{pmatrix} 1 \\ 5 \end{pmatrix} e^{3t}$$

The general solution is

$$\mathbf{x}(t) = c_1\mathbf{x}^{(1)}(t) + c_2\mathbf{x}^{(2)}(t) = c_1 \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + c_2 \begin{pmatrix} 1 \\ 5 \end{pmatrix} e^{3t}$$

Plug in initial value $\mathbf{x}(0) = (1,3)^T$ and solve for c_1 and c_2

$$c_1 = \frac{1}{2}, \quad c_2 = \frac{1}{2}$$

Thus the particular solution for the initial value problem is

$$\mathbf{x}(t) = \frac{1}{2} \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t} + \frac{1}{2} \begin{pmatrix} 1 \\ 5 \end{pmatrix} e^{3t}$$

As $t \to \infty$, the second term $\frac{1}{2}\begin{pmatrix}1\\5\end{pmatrix} e^{3t}$ is dominant and the first term $\frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix} e^{-t}$ is negligible. (It actually decays to $\mathbf{0}$.) $\mathbf{x}(t)$ becomes asymptotic to the line $x_2 = -5x_1$ as $t \to \infty$

7. Find the solution of the given initial value problem and describe the behavior of the solution as $t \to \infty$:

$$\mathbf{x}' = \begin{pmatrix}1 & -5\\1 & -3\end{pmatrix}\mathbf{x}, \quad \mathbf{x}(0) = \begin{pmatrix}1\\1\end{pmatrix}$$

Solution: At first, let us find the eigenvalues and eigenvectors of the coefficient matrix A as follows:

$$\det(A - r\mathbb{I}) = 0 \Rightarrow \begin{vmatrix}1-r & -5\\1 & -3-r\end{vmatrix} = 0 \Rightarrow r^2 + 2r + 2 = 0.$$

The solutions to the quadratic equation, or simply the eigenvalues are $r_{1,2} = -1 \pm i$. Next, the associated eigenvector $\boldsymbol{\xi}^{(1)} = \left(\xi_1^{(1)}, \xi_2^{(1)}\right)^T$ for $r_1 = -1 + i$ can be found via

$$\begin{pmatrix}1 & -5\\1 & -3\end{pmatrix}\begin{pmatrix}\xi_1^{(1)}\\ \xi_2^{(1)}\end{pmatrix} = (-1+i)\begin{pmatrix}\xi_1^{(1)}\\ \xi_2^{(1)}\end{pmatrix} \Rightarrow \begin{pmatrix}(2-i)\,\xi_1^{(1)} - 5\xi_2^{(1)}\\ \xi_1^{(1)} - (2+i)\,\xi_2^{(1)}\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}.$$

Solving the first equation, we arrive at

$$\xi_1^{(1)} = \frac{5}{2-i}\xi_2^{(1)},$$

or, for $\xi_2^{(1)} = (2-i)/5$ at

$$\boxed{\boldsymbol{\xi}^{(1)} = \begin{pmatrix}1\\ \frac{2-i}{5}\end{pmatrix}}. \tag{2}$$

On equally footing, and as per the second eigenvalue $r_2 = -1 - i$ we obtain

$$\begin{pmatrix}1 & -5\\1 & -3\end{pmatrix}\begin{pmatrix}\xi_1^{(2)}\\ \xi_2^{(2)}\end{pmatrix} = -(1+i)\begin{pmatrix}\xi_1^{(2)}\\ \xi_2^{(2)}\end{pmatrix} \Rightarrow \begin{pmatrix}(2+i)\,\xi_1^{(2)} - 5\xi_2^{(2)}\\ \xi_1^{(2)} - (2-i)\,\xi_2^{(2)}\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}.$$

From the first equation we get

$$\xi_1^{(2)} = \frac{5}{2+i}\xi_2^{(2)},$$

and for $\xi_2^{(2)} = (2+i)/5$ we obtain

$$\boxed{\boldsymbol{\xi}^{(2)} = \begin{pmatrix}1\\ \frac{2+i}{5}\end{pmatrix}}. \tag{3}$$

Hence, a fundamental set of solutions of the system (7) reads

$$\mathbf{x}^{(1)}(t) = \begin{pmatrix} 1 \\ \frac{2-i}{5} \end{pmatrix} e^{(-1+i)t}, \quad \mathbf{x}^{(2)}(t) = \begin{pmatrix} 1 \\ \frac{2+i}{5} \end{pmatrix} e^{-(1+i)t}.$$

Based on a well-known theorem, we can choose the real and imaginary parts of either $\mathbf{x}^{(1)}$ or $\mathbf{x}^{(2)}$ in order to construct a set of real-valued solutions. To that effect, let us pick $\mathbf{x}^{(1)}$ and obtain

$$\mathbf{x}^{(1)}(t) = \begin{pmatrix} 1 \\ \frac{2-i}{5} \end{pmatrix} e^{-t} \left(\cos(t) + i \sin(t)\right) = \mathbf{u}(t) + i\mathbf{v}(t), \tag{4}$$

with

$$\mathbf{u}(t) = \begin{pmatrix} e^{-t}\cos(t) \\ \frac{1}{5}e^{-t}\left(2\cos(t) + \sin(t)\right) \end{pmatrix}, \quad \mathbf{v}(t) = \begin{pmatrix} e^{-t}\sin(t) \\ \frac{1}{5}e^{-t}\left(2\sin(t) - \cos(t)\right) \end{pmatrix}. \tag{5}$$

This way, the general solution to the system (7) is given by

$$\mathbf{x} = \mathbf{x}(t) = c_1\mathbf{u} + c_2\mathbf{v}. \tag{6}$$

Upon employing the initial conditions provided, we end up with $c_1 = 1$ and $c_2 = -3$ such that the solution to the IVP (after performing the algebra) reads

$$\boxed{\mathbf{x} = \mathbf{x}(t) = e^{-t}\begin{pmatrix} \cos(t) - 3\sin(t) \\ \cos(t) - \sin(t) \end{pmatrix}}. \tag{7}$$

As $t \to \infty$, the solution will decrease in its amplitude since it exhibits exponential decay.

8. Let the system

$$\mathbf{x}' = \begin{pmatrix} \alpha & 1 \\ -1 & \alpha \end{pmatrix} \mathbf{x},$$

where α is a parameter.

(a) Determine the eigenvalues in terms of α.

(b) Find the bifurcation value or values of α where the qualitative nature of the phase portrait for the system changes.

(c) Draw a phase portrait for a value of α slightly below, and for another value slightly above, each bifurcation value.

Solution:

(a) At first, let us find the eigenvalues of the coefficient matrix as follows

$$\det(A - r\mathbf{I}) = 0 \Rightarrow \begin{vmatrix} \alpha - r & 1 \\ -1 & \alpha - r \end{vmatrix} = 0 \Rightarrow r^2 - 2\alpha r + \alpha^2 + 1 = 0,$$

where the roots of the latter equation are $\boxed{r_{1,2} = \alpha \pm i}$.

(b) The qualitative nature of the phase portrait for the system given will change when $\alpha = 0$. In particular, when the real part is negative, that implies that the origin is a spiral point and is asymptotically stable (since the trajectories approach it as t increases). On the other hand, if the real part is positive, the trajectories are expected to become unbounded, that is, the direction of motion is away from the origin.

(c) The phase portraits for the system given and for values of α of $\alpha = -0.5$ and $\alpha = 0.5$ are shown in the left and right panels of Fig. 1, respectively. Note that few solutions with various initial conditions (depicted with different colors) are shown too.

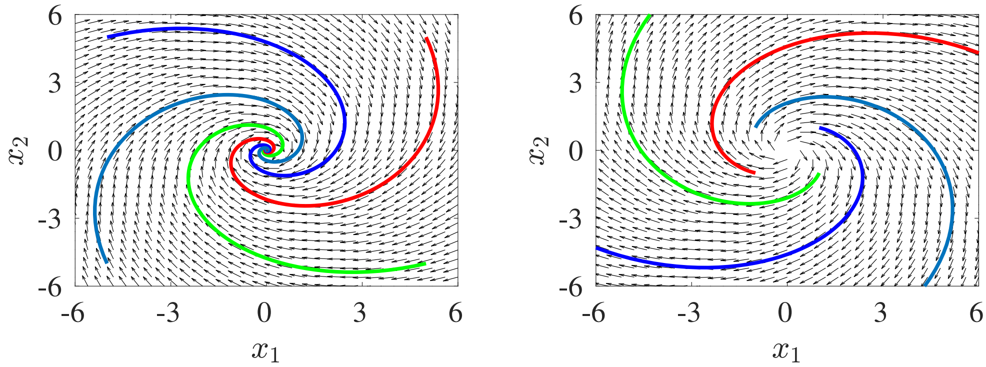

Figure 1: Phase portraits and solution curves for $\alpha = -0.5$ (left) and $\alpha = 0.5$ (right). Note that the arrows in the latter point outwards.