MATH 331 **<u>Final Exam</u>** **Spring 2018**

Name: ______________________________ **Student ID Number:** _______________

Instructor name: ___________________________ **Your section number** _________

Instructions

- **Turn off all cell phones and watch alarms!** Put away iPods, etc.
- There are 7 questions. Total is 100 points. Make sure that you have all of pages (9 pages in total).
- Do all work in this booklet. You may continue work to the backs of pages, but if you do so indicate where; make sure to write the final answer in the front page, and **circle your final answer**.
- You **may** use a calculator. If you do, be sure to show the set-up for what you are calculating and do **NOT** round intermediate results.
- Please write your work in an unambiguous order. Show all necessary steps or formulas.
- **Answers given without supporting work may receive 0 credit!**
- Be ready to show your UMass ID card when you hand in your Exam booklet.

QUESTION	PER CENT	SCORE
1	10	
2	15	
3	15	
4	15	
5	15	
6	15	
7	15	
TOTAL	100	

1. (10 points) Write the second order nonhomogeneous differential equation

$$y''(t) + 2\,y'(t) - 5\,y(t) = 9\sin t$$

as a first order system. Is the system linear or nonlinear? homogeneous or nonhomogeneous? Do not attempt to solve the system!

2. (15 points) Use the **Characteristic Equation** to find the general solution of the second-order homogeneous linear differential equation.

   (a) (7 points) $y''(t) - 2y'(t) + y(t) = 0$.

   (b) (8 points) $y''(t) - 2y'(t) + 2y(t) = 0$.

3. (15 points) Use the **Method of Undetermined Coefficients** to find the general solution of the second-order nonhomogeneous linear differential equation

$$y'' + 5\,y' + 4\,y = 10e^{-3t}$$

4. (15 points) Use the Laplace transform to solve the initial value problem

$$y'' + 9\,y = g(t), \quad y(0) = 0, \quad y'(0) = 0,$$

where the forcing function g is

$$g(t) = \begin{cases} 8\,\sin t & 0 \le t < \pi, \\ 0 & t \ge \pi. \end{cases}$$

5. (15 points) Use the Laplace transform to solve the initial value problem

$$y'' + 3\,y' + 2\,y = \delta(t-5), \quad y(0) = 0, \quad y'(0) = 1,$$

where δ is the Dirac δ-function.

6. (15 points) Let $A = \begin{pmatrix} 0 & 4 \\ 1 & 0 \end{pmatrix}$. Solve the initial value problem for the 2×2 system $\mathbf{x}' = A\mathbf{x}$ with initial condition $\mathbf{x}(0) = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$.

7. (15 points) Consider the first order linear ODE system: $\begin{cases} y_1'(t) = -y_1(t) + y_2(t) \\ y_2'(t) = -y_1(t) - y_2(t) \end{cases}$

(a) Compute the eigenvalues of the matrix corresponding to the system.

(b) Which of the following describe the critical point $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ of the system (circle all that apply):

[A] Proper Node [B] Saddle Point [C] Spiral Point [D] Center

[E] Stable [F] Unstable

(c) Without solving, describe the behavior of any solution $\mathbf{x}(t)$ satisfying $\mathbf{x}(0) \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ as $t \to \infty$.

(d) Draw a rough sketch of the phase portrait.

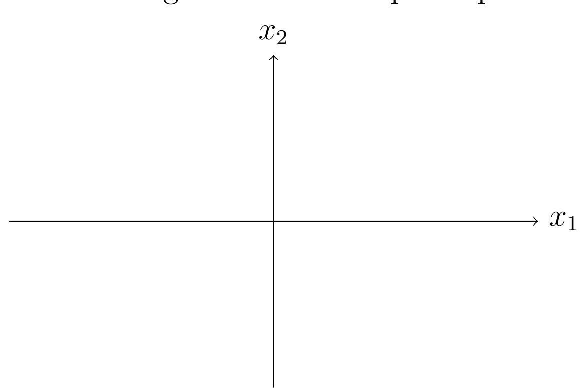

Table of Laplace Transform

$f(t)$	$\mathcal{L}(f(t))$		$f(t)$	$\mathcal{L}(f(t))$
1	$\frac{1}{s}$			
t	$\frac{1}{s^2}$			Derivatives
t^2	$\frac{2}{s^3}$		y	$\mathcal{L}(y)$
t^n	$\frac{n!}{s^{n+1}}$		y'	$s\mathcal{L}(y) - y(0)$
e^{at}	$\frac{1}{s-a}$		y''	$s^2\mathcal{L}(y) - sy(0) - y'(0)$
$t^n e^{at}$	$\frac{n!}{(s-a)^{n+1}}$			
$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$			
$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$			t-Shift
$\cosh(at)$	$\frac{s}{s^2-a^2}$		$f(t)$	$F(s)$
$\sinh(at)$	$\frac{a}{s^2-a^2}$		$u_a(t)f(t-a)$	$e^{-as}F(s)$
$e^{at}\cos(\omega t)$	$\frac{s-a}{(s-a)^2+\omega^2}$			
$e^{at}\sin(\omega t)$	$\frac{\omega}{(s-a)^2+\omega^2}$			s-Shift
$\delta(t-a)$	e^{-as}		$f(t)$	$F(s)$
$u_a(t)$	$\frac{e^{-as}}{s}$		$e^{at}f(t)$	$F(s-a)$

This page is a scratch paper.