Instructions

• Turn off all cell phones and watch alarms! Put away iPods, etc.

• Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate where.

• Do not use any other paper except this exam booklet and the one-page “cheat sheet” that you prepared. (Do not hand in your cheat sheet.)

• Organize your work in an unambiguous order. Show all necessary steps.

• Answers given without supporting work may receive 0 credit!

• If you use your calculator to do numerical calculations, be sure to show the setup leading to what you are calculating.

• Be prepared to show your UMass ID card when you hand in your exam booklet to your own instructor or TA as you exit the room.

<table>
<thead>
<tr>
<th>QUESTION</th>
<th>PER CENT</th>
<th>SCORE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
The printed exam will have 1 question per 1–2 pages with space for work.

1. (2 × 10% = 20%) The parts of this question are not related.
   (a) Evaluate the indefinite integral:
   \[ \int x e^{-2x} \, dx \]
   (b) Determine the derivative \( f'(x) \) of the function
   \[ f(x) = \int_{x}^{1} \frac{\sin \left( \sqrt{1 + t^2} \right)}{t} \, dt. \]

2. (2 × 10% = 20%)
   (a) Let \( R \) be the unbounded plane region in the first quadrant enclosed by the
   \( x \)-axis, the \( y \)-axis, and the graph of the function \( y = \frac{1}{1 + x^2} \). Compute the
   area of \( R \) by setting up and evaluating an appropriate improper integral.
   (Include in your work a rough sketch of the region \( R \).)
   (b) Now let \( S \) be the bounded plane region enclosed by the \( x \)-axis, the \( y \)-axis,
   the line \( x = 1 \), and the graph of that same function \( y = \frac{1}{1 + x^2} \). A solid
   is obtained by rotating \( S \) around the \( x \)-axis. Compute the volume of
   this solid by setting up and evaluating an appropriate definite integral.
   (Include in your work a sketch that shows a typical cross-section, disk,
   or washer consistent with the integral you set up.)

3. (2 × 10% = 20%) The parts of this question are not related.
   (a) Determine whether the series \( \sum_{n=2}^{\infty} \frac{(-1)^n}{n\ln n^{2/3}} \) is absolutely convergent,
       conditionally convergent, or divergent.
   (b) Find the interval of convergence of the power series \( \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x - 2)^n}{n4^n} \).

4. Curve \( C \) has polar equation \( r = \sin \theta + \cos \theta \).
   (a) (5%) Write parametric equations for the curve \( C \).
   \[
   \begin{cases}
   x = & \\
   y = &
   \end{cases}
   \]
   (b) (5%) Find the slope of the tangent line to \( C \) at its point where \( \theta = \frac{\pi}{2} \).
   (c) (10%) Calculate the length of the arc for \( 0 \leq \theta \leq \pi \) of that same curve
   \( C \) with polar equation \( r = \sin \theta + \cos \theta \).

5. (a) (12%) Determine the Taylor polynomial \( T_2(x) \) of degree 2 for the function
   \( f(x) = x^{1/7} \) centered at \( a = 1 \).
   (b) (8%) Suppose we were to use the approximation \( f(x) \approx T_2(x) \). Obtain
   an upper bound on the error of this approximation when \( 0.7 \leq x \leq 1.3 \).
   Give your answer rounded (up) to 4 decimal places.