Instructions

• **Turn off cell phones and watch alarms!** Put away cell phones, iPods, etc.

• There are six (6) questions.

• Do all work in this exam booklet. You may continue work to the backs of pages and the blank page at the end, but if you do so indicate where.

• Do not use any other paper except this exam booklet and the one-page “cheat sheet” that you prepared. (Do *not* hand in your cheat sheet.)

• Organize your work in an unambiguous order. Show all necessary steps.

• **Answers given without supporting work may receive 0 credit!**

• If you use your calculator to do numerical calculations, be sure to show the setup leading to what you are calculating.

• Be prepared to show your UMass ID card when you **hand in your exam booklet to your own instructor or TA as you exit the room.**

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The printed exam will have 1 question per 1–2 pages with space for work.

1. (2 × 8 = 16%) The parts of this question are not directly related!
   
   (a) If \( f(1) = 12 \), if the derivative \( f' \) is continuous, and if \( \int_1^4 f'(x) \, dx = 17 \), then what is the value \( f(4) \)?
   
   (b) Express the derivative \( g'(x) \), for \( 0 < x < \pi/2 \), as simply as possible if:
   \[
   g(x) = \int_{1/2}^{\sin x} \sqrt{1 - y^2} \, dy
   \]

2. (2 × 8% = 16%)
   
   (a) Calculate the area of the bounded region \( R \) enclosed by the curves
   \[
   y = x^3 + 4, \quad y = 4x^2 - 4x + 4.
   \]
   
   (b) The same region \( R \) as in (a)—enclosed by
   \[
   y = x^3 + 4, \quad y = 4x^2 - 4x + 4
   \]
   —is rotated around the \( x \)-axis. Express the volume of the resulting solid as an integral but do not actually evaluate that integral. And do not attempt to “simplify” the function inside the integral.

3. (3 × 6 = 18%) Use techniques of symbolic integration to evaluate:
   
   (a) \( \int x \, e^{-x} \, dx \)
   
   (b) \( \int \frac{x}{\sqrt{x^2 + \frac{9}{16}}} \, dx \)
   
   (c) \( \int \frac{x^2}{\sqrt{1 + x^2}} \, dx \)

4. A spiral has polar equation \( r = e^{-2\theta} \) for \( 0 \leq \theta < \infty \).
   
   (a) (6%) Write parametric equations for this spiral.
   
   (b) (10%) Find the length of the entire spiral for \( 0 \leq \theta < \infty \).
   \( \text{(Hint: This is easier to do if you work directly with the arc length formula for polar coordinates—and not the more general parametric formula.)} \)

5. (2 × 8 = 16%) Determine whether the series converges absolutely, converges conditionally only, or else diverges—and why.
   
   (a) \( \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3^n + \ln n} \)
(b) \[ \sum_{n=1}^{\infty} (-1)^n \frac{3^n}{n^2 \cdot 2^n} \]

6. (a) (6%) Starting with the Maclaurin series expansion of \( e^x \), express the function \( e^{-x^2} \) as the sum of a power series. Use summation (\( \sum \)) notation.

(b) (6%) Use (a) to express \( \int_0^{0.4} \frac{e^{-x^2} - 1}{x} \, dx \) as the sum of a series of numbers. Use \( \sum \) notation or give at least the first five terms of the series.

(c) (6%) What is the least number of terms of that numerical series you would need so as to approximate that integral with error magnitude less than \( 10^{-8} \)?

[When answering this question, do not actually make the approximation, and do not evaluate the integral from (b)!!]