(1) Define
$$Q(i) = \{a + bi \mid a, b \in \mathbb{Z}\}$$
where \(\mathbb{Z}\) denotes the integers. Show that \(Q(i)\) is a group with the usual addition law for complex numbers.

(2) Let \(Q(i)^\times\) denote the non-zero elements of \(Q(i)\); that is,
$$Q(i)^\times = \{a + bi \mid a, b \in \mathbb{Z} \text{ and not both } a \text{ and } b \text{ zero}\}.$$  
Show that \(Q(i)^\times\) is a group with the usual multiplication for complex numbers.

(3) Let \(GL_2(\mathbb{R})\) denote the set of \(2 \times 2\) matrices with real entries and non-zero determinant. Show that \(GL_2(\mathbb{R})\) is a group under matrix multiplication. (For associativity, you may just cite the fact that matrix multiplication is associative.) Why is the “non-zero determinant” condition important?

(4) Let \(SL_2(\mathbb{R})\) denote the set of \(2 \times 2\) matrices with real entries and determinant 1. Show that \(SL_2(\mathbb{R})\) is a group under matrix multiplication. (It may be helpful to know that the determinant is multiplicative: \(\det(AB) = \det(A) \det(B)\).)

(5) Let \(C_n\) denote the group of rotations of a regular \(n\)-gon. (In the group \(D_n\) we also allow reflections; here we do not.) Show that \(C_n\) is indeed a group under composition.

(6) Find the order of each element of \(C_7\) and \(C_8\).

(7) Find the order of each element of \(D_4\), the group of symmetries of a square.