Math 235 Practice Midterm 2

Q1.

(a) Compute the inverse of the matrix $A = \begin{bmatrix} 1 & 0 & -2 \\ 3 & 1 & -2 \\ -5 & -1 & 9 \end{bmatrix}$ by row reduction.

(b) Using your answer to part (a) or otherwise, solve the equation $A\mathbf{x} = \mathbf{b}$ where $\mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ -4 \end{bmatrix}$.

(c) Let A, B and C be invertible $n \times n$ matrices. Does the equation $A^{-1}(B - X)C = I_n$ have a solution X? If it has a solution, find it.

Q2.

(a) Let

$$T = \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix}.$$

Use row operations to show that $\det T = (b - a)(c - a)(c - b)$.

(b) Let

$$A = \begin{bmatrix} 1 & a & b + c \\ 1 & b & a + c \\ 1 & c & a + b \end{bmatrix}.$$

Find values for a, b, c such that the matrix A is invertible, or explain why no such numbers exist.

Q3.

(a) Find the volume of the parallelepiped P determined by the following three vectors in $\mathbb{R}^3$:

$$(1, 1, 1),\ (2, 3, 4),\ (1, 1, 5).$$

(b) Let $T\colon \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation given by

$$T(x_1, x_2, x_3) = (x_1 + x_2 - 2x_3, 3x_2 - x_3, 5x_3).$$

Compute the volume of the image $T(P)$ of the parallelepiped P from part (a) under the transformation T.

Q4.

(a) Let S be the set of vectors in $\mathbb{R}^4$ where the first and third coordinates are equal and the second coordinate is zero. Is S a subspace of $\mathbb{R}^4$? Justify your answer carefully.

(b) Consider the transformation on $\mathbb{P}_2$ given by $T(ax^2 + bx + c) = a - c$. Find polynomials that form a basis of the kernel of T.

Q5.

(a) Let $A = \begin{bmatrix} 5 & -3 & 2 \\ -1 & 1 & 3 \\ 4 & 1 & -1 \\ 1 & 2 & 2 \end{bmatrix}$.

(i) Let T_A be the linear transformation associated to A. Describe the kernel of T_A and give a basis for the kernel of T_A

(ii) Determine a basis for the column space of A.

(b) Let $M_{2\times 2}$ be the vector spaces of 2×2 matrices. Give coordinates of the matrix $\begin{bmatrix} 4 & 1 \\ 0 & 3 \end{bmatrix}$ in terms of the basis

$$\left\{ \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}, \begin{bmatrix} -1 & 2 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ -1 & 2 \end{bmatrix}, \begin{bmatrix} 3 & 1 \\ 0 & 1 \end{bmatrix} \right\}.$$

Q6. Let $\mathbb{P}_4$ denote the set of all polynomials in t of degree at most 4, and recall that $\mathbb{P}_4$ with the usual polynomial addition and scaling is a vector space.

(a) Let $H = \{f(t) \text{ in } \mathbb{P}_4 : f(t) - f(-t) = 0\}$. Find a basis of H. What is the dimension of H?

(b) Determine whether the set of polynomials

$$\mathcal{B} = \left\{ t - 2, t^2 - t + 4, t^2 + 1 \right\}$$

are linearly independent in $\mathbb{P}_4$.