Math 235 Practice Midterm 1

Q1. The following matrix is the augmented matrix for a system of linear equations.

$$A = \begin{bmatrix} 0 & 1 & 1 & 2 \\ 2 & 0 & 2 & -2 \\ -2 & -1 & -3 & 0 \\ 4 & 0 & 4 & -4 \end{bmatrix}$$

(a) Write down the linear system of equations whose augmented matrix is A.

(b) Find the reduced echelon form of A.

(c) In the reduced echelon form of A, mark the pivot positions.

(d) Does the system have no solutions, exactly one solution or infinitely many solutions? Justify your answer.

Q2.

(a) Describe the plane in $\mathbb{R}^3$ defined by the equation $7x_1 + 11x_2 + 13x_3 = 0$ as the span of a set of two vectors.

(b) Let A be a 5×3 matrix. Explain why the equation $A\mathbf{x} = \mathbf{b}$ cannot be consistent for all $\mathbf{b}$ in $\mathbb{R}^5$.

Q3. Consider the following nonhomogenous linear system:

$$\begin{aligned} x_1 + 3x_2 + 3x_3 &= 1 \\ 2x_1 + 6x_2 + 9x_3 &= 5 \\ -x_1 - 3x_2 + 3x_3 &= 5. \end{aligned}$$

(a) Find the general solution of the nonhomogenous linear system.

(b) Using your answer to part (a) or otherwise, find the general solution of the corresponding homogenous linear system.

Q4.

(a) Let $\mathbf{v}_1 = \begin{bmatrix} 4 \\ 10 \\ 6 \end{bmatrix}, \mathbf{v}_2 = \begin{bmatrix} 12 \\ 7 \\ 1 \end{bmatrix}, \mathbf{v}_3 = \begin{bmatrix} 2 \\ -5 \\ 3 \end{bmatrix}$. Determine whether $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ are linearly dependent or linearly independent, and give a justification.

(b) For the following statement, give an example or explain why no such example exists: a set of 5 vectors from $\mathbb{R}^3$ of which 4 are linearly independent.

(c) If the following statement is true, explain why and if false give a counterexample: every set of 3 distinct vectors in $\mathbb{R}^3$ contains *some* subset of 2 vectors that are linearly independent. In the latter case, explain why your example is a counterexample.

Q5.

(a) Find the standard matrix of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ such that $T(\mathbf{e}_1) = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$, $T(\mathbf{e}_2) = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, and $T(\mathbf{e}_3) = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$.

(b) Determine whether the transformation T from part (a) is onto.

(c) Determine whether the transformation T from part (a) is one-to-one

(d) Find the matrix associated to the geometric transformation on $\mathbb{R}^2$ that first reflects over the x-axis and then contracts in the x direction by a factor of $\frac{1}{2}$ and expands in the y direction by a factor of 3.

Q6.

(a) Let $S : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformation given by $S(\mathbf{x}) = B\mathbf{x}$ and let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by $T(\mathbf{x}) = A\mathbf{x}$, where

$$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} 0 & 1 & 2 \\ 2 & 0 & 1 \end{bmatrix}.$$

Find the standard matrix for the linear transformation $U : \mathbb{R}^3 \to \mathbb{R}^2$, where $U(\mathbf{x}) = T(S(\mathbf{x}))$.

(b) Consider the matrices

$$A = \begin{bmatrix} 2 & 2 & -2 \\ 5 & 1 & -3 \\ 1 & 5 & -3 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & -1 \\ 1 & 2 \\ -1 & 3 \end{bmatrix}.$$

(i) Compute the matrix power A^2.

(ii) Find the transpose B^T and evaluate the matrix multiplication $B^T A$.