Homework 9.1
Stat 605, Fall 2018

Instructor: Brian Van Koten

Due Thursday, November 15

Here is one last problem using the CLT:

Problem 1. *Let* $\{X_j\}_{j=1}^{\infty}$ *be i.i.d. random variables having the double exponential distribution with parameter* 1*, i.e. the common density is* $f(x) = \exp(-|x|)/2$*. Show that*

$$\sqrt{N}\left(\frac{\sum_{j=1}^{N} X_j}{\sum_{j=1}^{N} X_j^2}\right) \xrightarrow{\mathscr{D}} N\left(0, \frac{1}{2}\right).$$

Hint: Use Slutsky's theorem.

Problem 2 (3+3 points)**.** *Let* X *and* Y *be* $\mathbb{R}$*-valued random variables with joint density* $f_{X,Y}$*. Let* f_X *and* f_Y *denote the marginal densities of* X *and* Y*, respectively.*

1. *Prove that for any Borel set* B*,*

$$\mathbb{P}[Y \in B | X = x] = \int_{y \in B} \frac{f_{X,Y}(x, y)}{f_X(x)}\, dy$$

   for $\mathbb{P}_X$*-almost all* $x \in \mathbb{R}$*.*

2. *Suppose that* $\mathbb{E}[|Y|] < \infty$*. Prove that*

$$\int y \frac{f_{X,Y}(x, y)}{f_X(x)}\, dy$$

   is one version of $\mathbb{E}[Y|X]$*.*