Homework 7.1
Stat 605, Fall 2018

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Due Tuesday, October 29

Our current notation for the distribution of a random variable makes a reference to the underlying probability space: For $X : (\Omega, \mathscr{F}, \mathbb{P}) \to (\mathbb{R}, \mathscr{B})$, we write $\mathbb{P}_X$ for the distribution of X. This notation may introduce unnecessary difficulties when the random variables involved in a problem come from different probability spaces. One solution is to let $\mathcal{L}(X)$ denote the distribution of the random variable X. Here, $\mathcal{L}(X)$ is pronounced "law of X."

Problem 1 (5 points). *Let δ_c be the distribution of a constant random variable taking the value $c \in \mathbb{R}$. That is, let δ_c be the Borel measure defined by*

$$\delta_c(A) = \begin{cases} 1 & \text{if } c \in A, \\ 0 & \text{if } c \in A^c. \end{cases}$$

Let $\{X_n\}_{n=1}^{\infty}$ be a sequence of random variables such that $\mathcal{L}(X_n) \Rightarrow \delta_c$. Show that $X_n \xrightarrow{\text{P}} c$. Note: *We do not need to assume that $\{X_n\}_{n=1}^{\infty}$ are defined on the same probability space. In this problem, since the limit is a constant, we define $X_n \xrightarrow{\text{P}} c$ to mean that for every $\varepsilon > 0$,*

$$\lim_{n \to \infty} \mathbb{P}[|X_n - c| > \varepsilon] = 0.$$

Problem 2 (2+1+3 points). *Let Y be a real valued random variable with $\mathcal{L}(Y) = \mu$, where μ has continuous density f. For any $n \in \mathbb{N}$, let $Y_n = \lfloor nY \rfloor / n$, and let $\mu_n = \mathcal{L}(Y_n)$.*

1. *Describe μ_n explicitly.*
2. *Does μ_n have a density for any n?*
3. *Prove that $\mu_n \Rightarrow \mu$.*