Homework 5.2
Stat 605, Fall 2018

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Due Thursday, October 11

Problem 1 (0 points)**.** *Read the statement and proof of Jensen's inequality, Proposition 5.1.4 in Rosenthal's book. If you want more details, read the proof of Theorem 1.5.1 in Durrett's book. (Recall that Durrett's book is online; there is a link on the course web page.) You don't have to turn anything in related to this problem, but you should use Jensen's inequality to do the first problem in the assignment given on Tuesday.*

Problem 2 (3 points)**.** *Let* $\{X_i\}_{i=1}^{\infty}$ *be independent random variables. Show that* $\sup_n X_n < \infty$ *if and only if* $\sum_{n=1}^{\infty} \mathbb{P}[X_n > A] < \infty$ *for some* A*.*

Problem 3 (3 points)**.** *Prove* Cantelli's inequality*: If* X *is a random variable with finite mean and variance, then*

$$\mathbb{P}[X - m \geq \alpha] \leq \frac{v}{v + \alpha^2}.$$

Hint: *First, show that* $\mathbb{P}[X - m \geq \alpha] \leq \mathbb{P}[(X - m + y)^2 \geq (\alpha + y)^2]$*. Then use Markov's inequality and minimize over* $y > 0$*.*

Problem 4 (3 points)**.** *Prove that if* $X_n \overset{\text{as}}{\to} X$*, then for any* $\varepsilon > 0$*, we have* $\mathbb{P}[|X_n - X| > \varepsilon \textit{ i.o.}] = 0$*.* Note: *This is the converse of the statement which we used in the proof of the strong law of large numbers and the proof that convergence in probability implies almost sure convergence of a subsequence. To make sure that you understand this point, read Lemma 5.2.1 and its proof in Rosenthal's book.*