Homework 5.1
Stat 605, Fall 2018

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Due Thursday, October 11

The following problem explains why we care about the moment condition in the law of large numbers: $X_i \in L^4$ is a stronger condition than $X_i \in L^1$.

Problem 1 (2+2 points)**.** *Let* $1 \le p < q < \infty$*.*

1. *Let* $X : (\Omega, \mathscr{F}, \mathbb{P}) \to (\mathbb{R}, \mathscr{B})$ *be a random variable. Show that if* $X \in L^q$*, then* $X \in L^p$*.*
2. *Give an example of a random variable* X *on some probability space so that* $X \in L^p$*, but* $X \notin L^q$*.*

Note: *For general measures (not necesarily probability distributions), there are no inclusions among the various* L^p *spaces.*

Problem 2 (4 points)**.** *Let* $\delta, \varepsilon > 0$*, and let* $\{X_i\}_{i=1}^{\infty}$ *be a sequence of independent nonnegative random variables defined jointly on some probability space such that*

$$\mathbb{P}[X_i \ge \delta] \ge \varepsilon$$

for all $i \in \mathbb{N}$*. Prove that with probability one,*

$$\sum_{i=1}^{\infty} X_i = \infty.$$

Problem 3 (4 points)**.** *Let* $\{X_i\}_{i=1}^{\infty}$ *be random variables defined jointly on some probability space with* $\mathbb{E}[X_i] = 0$ *and* $\mathbb{E}[X_i^2] = 1$*. Prove that* $\mathbb{P}[X_n \ge n \text{ i.o.}] = 0$*.*