Homework 2.1
Stat 697U, Spring 2019

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Due Wednesday, February 6

The following is a variation of a problem in *Markov Chains* by Norris.

Problem 1 (1+3+3 points). *A gambler begins with* \$2 *and needs to win* \$10 *in a hurry. She can play the following game: A fair coin is tossed. If a player bets on the correct side, she wins an amount equal to her bet. Otherwise, she loses her bet. The gambler decides to play the game by the following strategy: She bets her entire fortune when she has less than* \$5*, and if she has more than* \$5 *she bets just enough to increase her fortune to* \$10*. The gambler stops if her fortune reaches* \$0 *or* \$10*.*

1. *Write down the initial distribution and transition probability matrix of a homogeneous Markov chain* X_n *modeling the gambler's fortune after* n *coin flips. Draw the transition graph corresponding to this chain.*
2. *What is the probability that the gambler's fortune reaches* \$10*?*

The following problem comes from Bremaud's book:

Problem 2 (3 points). *The Markov property does not imply that the past and future are independent given* any *information concerning the present. Give an example of a HMC* X_n *with state space* $\{1, 2, 3, 4, 5, 6\}$ *so that*

$$\mathbb{P}[X_2 = 6 | X_1 \in \{3, 4\}, X_0 = 2] \neq \mathbb{P}[X_2 = 6 | X_1 \in \{3, 4\}].$$