PUTNAM 2010 WEEK 0: BRAINSTORMING PROBLEMS

1. A census-taker knocks on a door, and asks the woman inside how many children she has and how old they are.

"I have three daughters, their ages are whole numbers, and the product of their ages is 36."

"That's not enough information," responds the census-taker.

"I'd tell you the sum of their ages, but you'd still be stumped."

"I wish you'd tell me something more."

"Okay, my oldest daughter Annie likes dogs."

What are the ages of the three daughters?

2. Given a picture of 100 circles, prove that it is possible to color regions in black and white so that any two regions that have a common border are colored differently. Here is an example:

3. For each integer $n > 1$, find distinct positive integers x and y such that
$$\frac{1}{n} = \frac{1}{x} + \frac{1}{y}$$

4. Find the maximal number of regions into which 2010 lines can divide the plane.

5. Lockers in a row are numbered $1, 2, 3, \ldots, 1000$. At first, all the lockers are closed. A person walks by and opens every other locker, starting with locker #2. Thus lockers $2, 4, 6, \ldots, 998, 1000$ are now open. Another person walks by, and changes the "state" (i.e., closes a locker if it is open, opens a locker if it is closed) of every third locker, starting with locker #3. Then another person changes the state of every fourth locker, starting with #4, etc. This process continues until no more lockers can be altered. Which lockers will be closed?