Mathematics in the late 16th - early 17th century

• Simon Stevin (1548 – 1620) wrote a short pamphlet *La Disme*, where he introduced decimal fractions to a wide audience.

• Transition to Hindu-Arabic from Roman numerals was initially resisted, hence the practice of writing amounts on checks.
Beginnings of Calculus

- Archimedes (287 BC - 212 BC) “The Quadrature of the Parabola” area under the curve using “Riemann sums”

- Kepler (1571-1630) “New solid geometry of wine bottles” - various optimization problems

- Napier (1550-1617) “Description of the wonderful canon of logarithms” - Laplace said that invention of logarithms “by shortening the labors, doubled the life of an astronomer”

- Descartes (1596 – 1650) “Discourse on the Method for Rightly Directing One's Reason and Searching for Truth in the Sciences” - Analytic Geometry (study of geometry using algebra and Cartesian coordinate system)
• Pascal (1623–1662) “Treatise on the sines of the quadrant” - area under the sine curve - integral of $\sin(x)$
• Gregory (1584-1667) - integral of $1/x$ is equal to $\ln(x)$
• Van Heuraut (1634-1660) - “Epistola de transmutatione curvarum linearum in rectas” - arc length formula
• Fermat (1601-1665)
  • “Methodus ad disquirendam maximam et minima” - Fermat’s principle - in modern day’s formulation, if $f(x)$ achieves extremum at $x=a$ then $f''(a)=0$
  • “De tangentibus linearum curvarum” - construction of tangents to curves
Barrow (1630-1677) “Geometrical Lectures” - Fundamental Theorem of Calculus. He discovered both its geometric manifestation - relationship between finding tangents (“derivatives”) and the area (“antiderivatives”) and the physical interpretation - relationship between computing velocity (derivatives) and distance (integrals).

His father decided that he can’t become a successful merchant because of his temper and sent him to study theology (!), which in turn get him interested in chronology, then astronomy, then geometry (to construct lenses for the telescope).

On a journey to the Holy Land, the ship was attacked by pirates. He was the only passenger to join the crew sword in hand to repel the pirates.

Barrow took a very prestigious position as a Lucasian professor of geometry at Cambridge, where one of his students was Isaac Newton (who later took the same position and Barrow happily became a full-time theologian.)
• It looks like everything that modern students study in Calculus was discovered in the 17th century before Newton.
• Why is he called an inventor of Calculus?
Newton discovered a major method to solve any equation, algebraic or differential (the concept he introduced) using power series, also known as Taylor series (Taylor was a student of Newton).

He was guided by an analogy:

<table>
<thead>
<tr>
<th>numbers</th>
<th>decimal fractions</th>
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<tbody>
<tr>
<td>functions</td>
<td>power series</td>
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Fluxions and Fluents

• For Newton, every variable $x$ he considered depended on time $t$. Nowadays we would use a function notation $x=f(t)$. Newton called the quantity $x$ the **fluent**.

• The **fluxion** is the speed with which $x$ increases. In modern notation, this is $f'(t)$, although Newton’s notation is still used, especially in physics.
Newton applied his new mathematics to solve a staggering number of mathematical and physical problems, many of them contained in “Principia” (Mathematical Principles of Natural Philosophy). The crowning achievement of Principia was introduction of the Law of Gravity and demonstration that

- Law of Gravity implies Laws of Kepler (easy, see Calc III).
- Law of Gravity is the only possible central force field where Laws of Kepler can hold (this is a difficult theorem)