1. Al-Khwarizmi gives the following rule for solving his sixth case of the quadratic equation, \( bx + c = x^2 \): Halve the number of roots. Multiply this by itself. Add this square to the number. Extract the square root. Add this to the half of the number of roots. That is the solution.

(a) Translate this rule into a formula.
(b) Give a geometric argument for its validity using the following figure, where \( x = AB, b = HC, c \) is the area of the rectangle \( ABRH \), and \( G \) is the midpoint of \( HC \).

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2. This problem is from Fibonacci’s Liber abbaci. Four men already having denari found a purse of denari; the first man said that if he would have the denari from the purse, then he would have twice as many as the second. The second, if he would have the purse, then would have three times as many as the third, and the third, if he would have it, then he would have four times as many as the fourth. The fourth, five times as many as the first. How much denari does each man have?

3. Another problem from Liber abbaci. There is a vat that has four holes, and by the first hole the vat can be emptied in 1 day, by the second in 2, by the third in 3, and by the fourth in 4; it is sought in how many hours the vat will be emptied if the said four holes are opened together.

4. This problem is from the Treviso Arithmetic, the first printed arithmetic text, dated 1478: The Holy Father sent a courier from Rome to Venice, commanding him that he should reach Venice in 7 days. And the most illustrious Signoria of Venice also sent another courier to Rome, who should reach
Rome in 9 days. And from Rome to Venice is 250 miles. It happened that by order of these lords the couriers started their journeys at the same time. It is required to find in how many days they will meet.

5. Express $\sqrt{27} + \sqrt{200}$ as $a + \sqrt{b}$ with integers $a, b$. This problem is from Rudolf’s Coss (the word coss in the title is the German form of the Italian cosa, or thing, the name given to the unknown in an algebraic equation).

6. Translate into modern algebraic language a geometric proof by al-Karaji (see section 9.3) of the formula

$$\sum_{i=1}^{n} i^3 = \left( \sum_{i=1}^{n} i \right)^2.$$

7. Show that if $a, b, c$ are roots of the equation $x^3 + \alpha x^2 + \beta x + \gamma = 0$ then

$$a + b + c = -\alpha,$$
$$ab + ac + bc = \beta,$$
$$abc = -\gamma.$$  

These formulas were discovered by Francois Vieté.

8. Use Tartaglia’s method to solve the cubic equation $x^3 + 3x = 10$.

9. Use Ferrari’s method to solve the quartic equation $x^4 + 4x + 8 = 10x^2$ (see section 12.3 for the method).

10. Write a mini-essay (typed one page) about establishing the status of a mathematician in different eras. Specifically, compare ancient Greece, 16th century Italy, and modern academia.