#1

Diagonals: \( \overrightarrow{AC} = \langle -1, 1, 1 \rangle \)
\( \overrightarrow{DB} = \langle 1, -1, 1 \rangle \)

Angle between vectors \( \cos \theta = \frac{\overrightarrow{AC} \cdot \overrightarrow{DB}}{|\overrightarrow{AC}| \cdot |\overrightarrow{DB}|} \)

\[
= \frac{-1-1+1}{\sqrt{3} \sqrt{3}} = \frac{-1}{3} < 0 \quad \text{obtuse angle}
\]

Acute angle \( \cos \frac{1}{3} \approx 70.5 \) degrees

#2

We are given that spheres intersect in a circle but to be on a safe side let's check that:

Radii are \( \sqrt{44} \) and \( \sqrt{29} \)

And distance between the centers is:

\[
\sqrt{(1-0)^2 + (2-0)^2 + (1-1)^2} = \sqrt{5} < \sqrt{44} + \sqrt{29}
\]

The circle of intersection is obtained by rotating \( P \) around the line \( 0, 02 \).
Along this circle both equations
\[
\begin{align*}
(x-1)^2 + (y-2)^2 + (z+1)^2 &= 44 \\
(x-3)^2 + (y-2)^2 + z^2 &= 29
\end{align*}
\]
are satisfied, and therefore also their difference:
\[
(x-1)^2 + (y-2)^2 + (z+1)^2 - (x-3)^2 - (y-2)^2 - z^2 = 44 - 29
\]
\[
x - 2x + 1 + z^2 + 2z + 1 - x^2 + 6x - 9 - z^2 = 15
\]
\[
4x + 2z = 22
\]
\[
2x + z = 11
\]
This is linear equation, and so gives the place. This plane contains one circle of intersection. So it's the place we were looking for

\[
2x + z = 11
\]
Its direction vector is perpendicular to \(<2, 1, 2>\) and \(<1, 2, 3>\), so we can compute it as the cross-product:

$$\vec{d} = \langle 2, 1, 2 \rangle \times \langle 1, 2, 3 \rangle = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = \langle -1, -4, 3 \rangle$$

It remains to find a point on \(L\). For example, let's find the point of intersection of \(L\) and \(L_2\). To do this, let's find the equation of the plane which contains \(L\) and \(L_2\).

It passes through the point \((3, 4, 5)\) and has normal vector \(\vec{d} = \langle 1, 2, 3 \rangle\):

$$\vec{d} \times \langle 1, 2, 3 \rangle = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -1 & 4 & 2 \end{vmatrix} = \langle -18, 6, -2 \rangle$$
So the equation of S is
\[-18(x-3) + 6(y-4) + 2(z-5) = 0\]
\[-9(x-3) + 3(y-4) + (z-5) = 0\]
\[-9x + 27 + 3y - 12 + z - 5 = 0\]
\[-9x + 3y + z + 10 = 0\]

Great. So now let's intersect this plane S with the line L, which has parametric equation

\[
\begin{align*}
  x &= 1 + 2t \\
  y &= -1 + t \\
  z &= 2t 
\end{align*}
\]

We get

\[-9(1+2t) + 3(-1+t) + 2t + 10 = 0\]
\[-9 - 18t - 3 + 3t + 2t + 10 = 0\]
\[-13t - 2 = 0\]
\[t = -\frac{2}{13}\]
So P has coordinates
\[ (1 - \frac{4}{13}, -1 - \frac{2}{13}, -\frac{4}{13}) = (\frac{9}{13}, \frac{-15}{13}, -\frac{4}{13}) \]

Finally, the symmetric equation of the line is:

\[ \frac{x - \frac{9}{13}}{-1} = \frac{y + \frac{15}{13}}{-4} = \frac{z + \frac{4}{13}}{-3} \]

#4  \(x^2 + 2x = y^2 + 2z^2\)

\((x+1)^2 = y^2 + 2z^2 + 1\)

Hyperboloid of 2 sheets

Intercepts:
- X-axis:  \(y = z = 0\)
  \(x = 0\) or \(x = -2\)
- Y-axis:  \(x = t = 0 \rightarrow y = 20\)
- Z-axis:  \(x = y = 0 \rightarrow z = 20\)
\#5

\[ \mathbf{P}(t) = \langle \ln(t-2), 2t^2 - t, t+1 \rangle \]
\[ \mathbf{V}(t) = \langle \frac{1}{t-2}, 6t^2 - 1, 1 \rangle \]
\[ \mathbf{a}(t) = \langle -\frac{1}{(t-2)^2}, 12t + 10 \rangle \]
\[ \mathbf{a}(t) \times \mathbf{V}(t) = \begin{vmatrix} i & j & k \\ \frac{1}{(t-2)^2} & 12t + 10 & 0 \\ \frac{1}{t-2} & 6t^2 - 1 & 1 \end{vmatrix} = \langle \frac{1}{(t-2)^2}, -\frac{6t^2 - 1}{(t-2)^2}, -\frac{12t}{t-1} \rangle \]

\[ \mathbf{a}(3) = \langle -1, 36, 0 \rangle \]
\[ \mathbf{V}(3) = \langle 1, 53, 1 \rangle \]

\[ \text{Comp}_{\mathbf{V}(3)} \mathbf{a}(3) = \frac{\mathbf{a}(3) \cdot \mathbf{V}(3)}{\| \mathbf{V}(3) \|} = \frac{-1 + 36 \cdot 53}{\sqrt{1 + 53^2 + 1}} = \frac{1907}{\sqrt{2811}} \]
\[ \int_{0}^{1} e^{t} \left( 1 + (\sin t + \cos t)^2 + (\cos t - \sin t)^2 \right) dt = 0 \int_{0}^{1} \sqrt{1 + 1 + 1} dt = 0 \int_{0}^{1} \sqrt{3} e^{t} dt = \left[ \sqrt{3} e^{t} \right]_{0}^{1} = \sqrt{3} e - \sqrt{3} \]