STAT 515

Homework 8

Each question is worth 4 points and will be combined with your score on the webwork assignment. Please write out your answers and turn them in at the beginning of class on the due date. Show work to receive full credit. You may work on these problems with others but your answers must be your own. Please note the names of anyone that you worked with at the end of the homework.

1. Find the distributions of the random variables that have the mgf.s[1]:
   (a) $\psi_X(t) = \left[(1/3)e^t + (2/3)\right]^5$.
   (b) $\psi_Y(t) = \frac{e^t}{2-e^t}$.
   (c) $\psi_Z(t) = e^{2(e^t-1)}$.
2. Suppose that $Y \sim \text{Binomial}(n, p)$ and let $Y^\star = n - Y$.
   (a) Show that $\mathbf{E}(Y^\star) = nq$ and $\text{Var}(Y^\star) = npq$ where $q = 1 - p$.
   (b) Show that the mgf of $Y^\star$ is $\psi_\star(t) = (qe^t + p)^n$ where $q = 1 - p$.
   (c) Based on your answer to part 2b, what is the distribution of $Y^\star$?
   (d) If Y is interpreted as the number of successes in a sample of size n, what is the interpretation of $Y^\star$?
   (e) Based on your answer in part 2d, why are the answers to the first three parts of question 2 "obvious"?
3. (a) Show that if Y is a random variable with mgf $\psi(t)$ and U is given by $U = aY + b$, for constants a and b, then the mgf of U is $\psi_U(t) = e^{tb}\psi(at)$.
   (b) If Y is a random variable with mean μ and variance σ^2, use the mgf of U to derive the mean and variance of U.
4. A random variable Y has the density function $f(y) = e^y$ for $y < 0$ (and is 0 elsewhere).
   (a) Find $\mathbf{E}(e^{3Y/2})$.
   (b) Find the mgf for Y.
   (c) Find $\text{Var}(Y)$.
5. Suppose that Y is uniformly distributed on $(0, 1)$ and that $a > 0$ is constant.
   (a) Give the mgf for Y.
   (b) Derive the mgf for $W = aY$. What is the distribution of W? Why?
   (c) Derive the mgf of $X = -aY$. What is the distirbution of X? Why?
   (d) If b is a fixed constant, derive the mgf of $V = aY + b$. What is the distribution of V? Why?

[1] moment-generating functions