1. Today you have 100,000 in a bank which earns an effective annual interest rate of 5%. You wish to convert this to a perpetuity due giving annual payments, starting at K and increasing 2% each year. Compute K.

The answer is 2857, 2877, 2897, 2917, or 2937.

Solution:

\[
K \sum_{k=0}^{\infty} \frac{1.02^k}{1.05} = \frac{1}{1 - \frac{1.02}{1.05}} = 100,000. \quad K = 2857
\]

2. An account earns interest at a nominal annual rate of 6%, compounded monthly, from the start of year 0 until the end of year 5. After that, the accounts earns at a force of interest of \( \delta(t) = \sqrt{t/50} \) until the end of year 10. Here \( t \) measures in years the time from year 0. Compute the nominal annual interest rate \( Y \), compounded semi-annually, the account earns over the 10 year period.

The answer is 5.4, 5.6, 5.8, 6.0, 6.2%.

Solution: Suppose you start with 1000. Then the end balance \( BAL \) is

\[
BAL = 1000\left(1 + \frac{0.06}{12}\right)^{5\times12} \exp\left(\int_{5}^{10} \frac{t^{1/2}}{50} dt\right)
\]

\[
= 1348.85 \times \exp\left((10^{3/2} - 5^{3/2})/(3/2)/50\right)
\]

\[
= 1771.49
\]

So

\[
1771.49 = 1000\left(1 + \frac{Y}{2}\right)^{10\times2}. \quad Y = 0.058 = 5.8%
\]

3. A bank customer takes out a loan of 500 with 16% nominal annual interest rate convertible quarterly. The customer makes a payment of 20 at the end of each quarter. Calculate the amount of principal in the fourth payment.

The answer is 0.0, 0.9, 2.7, 5.2, or “There is not enough information to calculate the amount of principal.”

Solution: This is Module 3 #11 (May 05 #25).
4. Assume every year has 365 days and every month has \(365/12 = 30.4\) days. A 1000 par bond with 6% semiannual coupons matures in \(1760 = 4 \times 365 + 300\) days. If the bond is priced to yield an effective annual rate of 5.4%, compute this purchase price (also known as price+accrued). Assume (as in Module 4) simple interest between coupon dates.

The answer is 1000, 1020, 1040, 1060, or 1080.

**Solution:** Price+accrued (PPA) is the price \(P\) at the last settlement date (65 days ago), with interest accruing at 5.4% effective annual interest rate. First we find that the nominal annual bond yield is 5.33% compounded semiannual. To compute \(P\), set \(N = 5 \times 2, I/Y = 5.33/2, PMT = 0.06/2 \times 1000, FV = 1000\) and compute \(PV = -1029\). So

\[
PPA = 1029(1 + 0.054)^{65/365} = 1039.5
\]

5. Mike deposits $100 at the end of every 3 months into an account for 30 years. The account earns a nominal annual interest of 5% per annum compounded quarterly. Ten years after Mike’s first deposit, Mia makes quarterly deposits for twenty years into another account earning nominal annual interest of 6% compounded quarterly. Mia’s first deposit is $M$ and her deposits increase by $10$ every quarter. If Mia has twice as much as Mike at the time of their last deposit, compute \(M\).

The answer is 43, 53, 63, 73, 83.

**Solution:** Let \(a_{n;i}\) denote the present value of an annuity where 1 is deposited every unit of time for \(n\) units of time, earning interest \(i\) over each unit of time. Let \(s_{n;i}\) denote the future value of this. Twice the future value of Mike’s savings is

\[
2 \times 100 \times s_{4 \times 30;5/4} = 200 \times 275.22 = 55042.41.
\]

The future value of Mia’s savings is

\[
(1.015)^{4 \times 20} \left(Ma_{4 \times 20;6/4} + 10 \left(\frac{a_{4 \times 20;6/4} - 80 \times 1.015^{-80}}{0.015}\right)\right) = 152.71M + 48457.8
\]

So \(M = (55042.410 - 48457.8)/152.71 = 43\).

6. A 5-year 1000 par value 6% bond pays coupons semi-annually. The bond is priced at 950. By how much should the redemption increase in order to increase the bond price to 1000?

The answer is 11, 31, 53, 71, or 91.

**Solution:** Set \(PV = -950, FV = 1000, N = 10, PMT = 30\) and compute I/Y to be 3.60. Then change \(PV = 1000\) and compute \(FV = 1071\).

7. The Hampshire organic farm project requires an initial investment of 2500 today. It will pay 500 at the end of years 1, 2, and 3, and 1000 at the end of years 4 and 5. The Amherst free range farm project requires an initial investment of 1500 today.
It will pay $X$ in one year and $1500$ in two years. The net present values of the two projects are equal at an interest rate of $10\%$. Compute $X$.

The answer is $308.46$, $318.46$, $328.46$, $338.46$, or $348.46$.

**Solution:**

Under the CF menu set $C_{F0} = -2500$, $C01 = 500$, $F01 = 3$, $C02 = 1000$, $F02 = 2$, and under the NPV menu set $I = 10$ and we compute $NPV = 47.36$ for the NPV of Hampshire project. The NPV of Amherst project therefore satisfies

$$47.36 = -1500 + X \times 1.10^{-1} + 1500 \times 1.10^{-2}.$$

$X = 338.46$